VSSUP, Jan 2014

Quantum optomechanics

[Quantum Langevin approach to atom optics-inspired techniques]

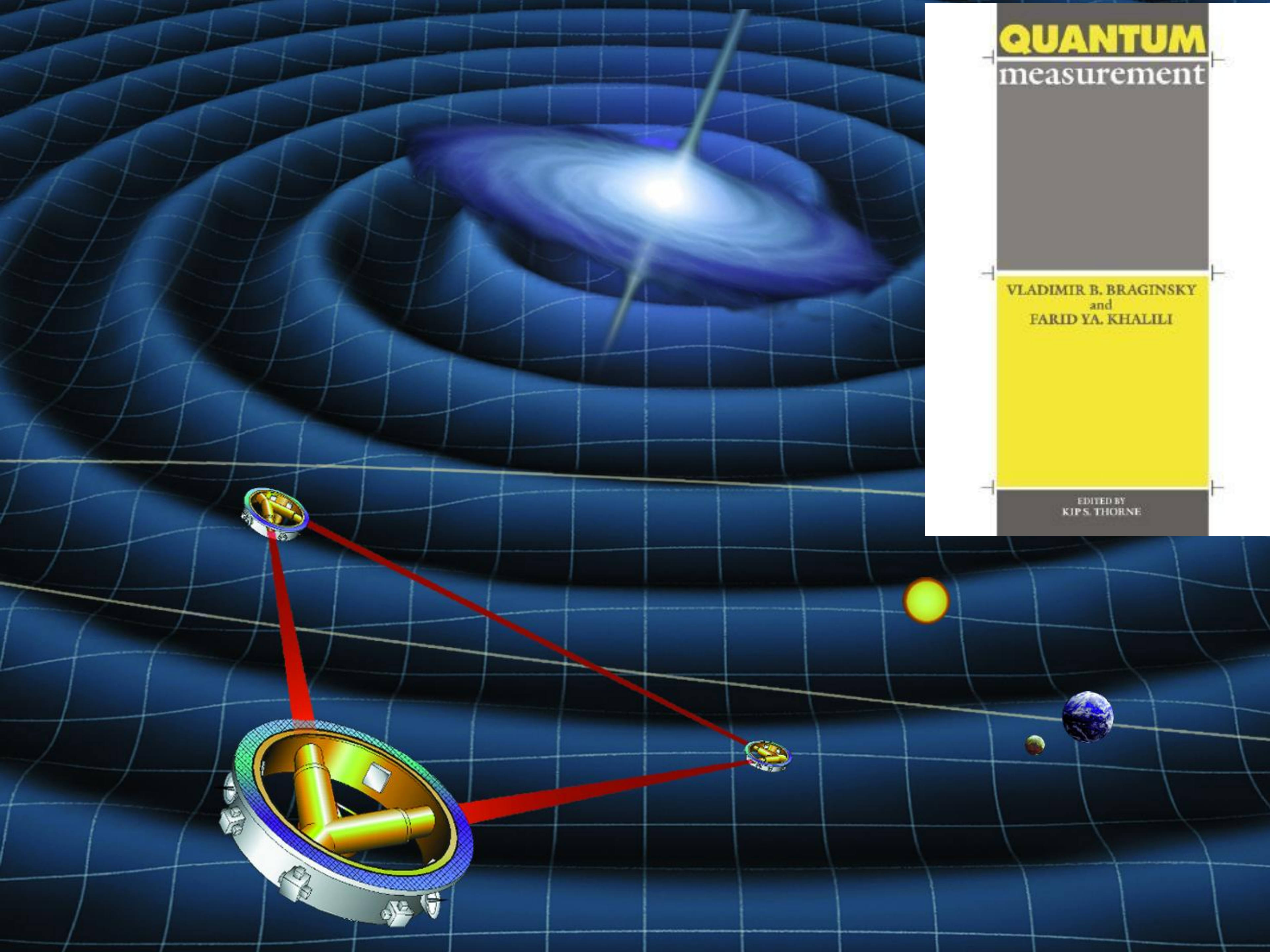
Warwick Bowen

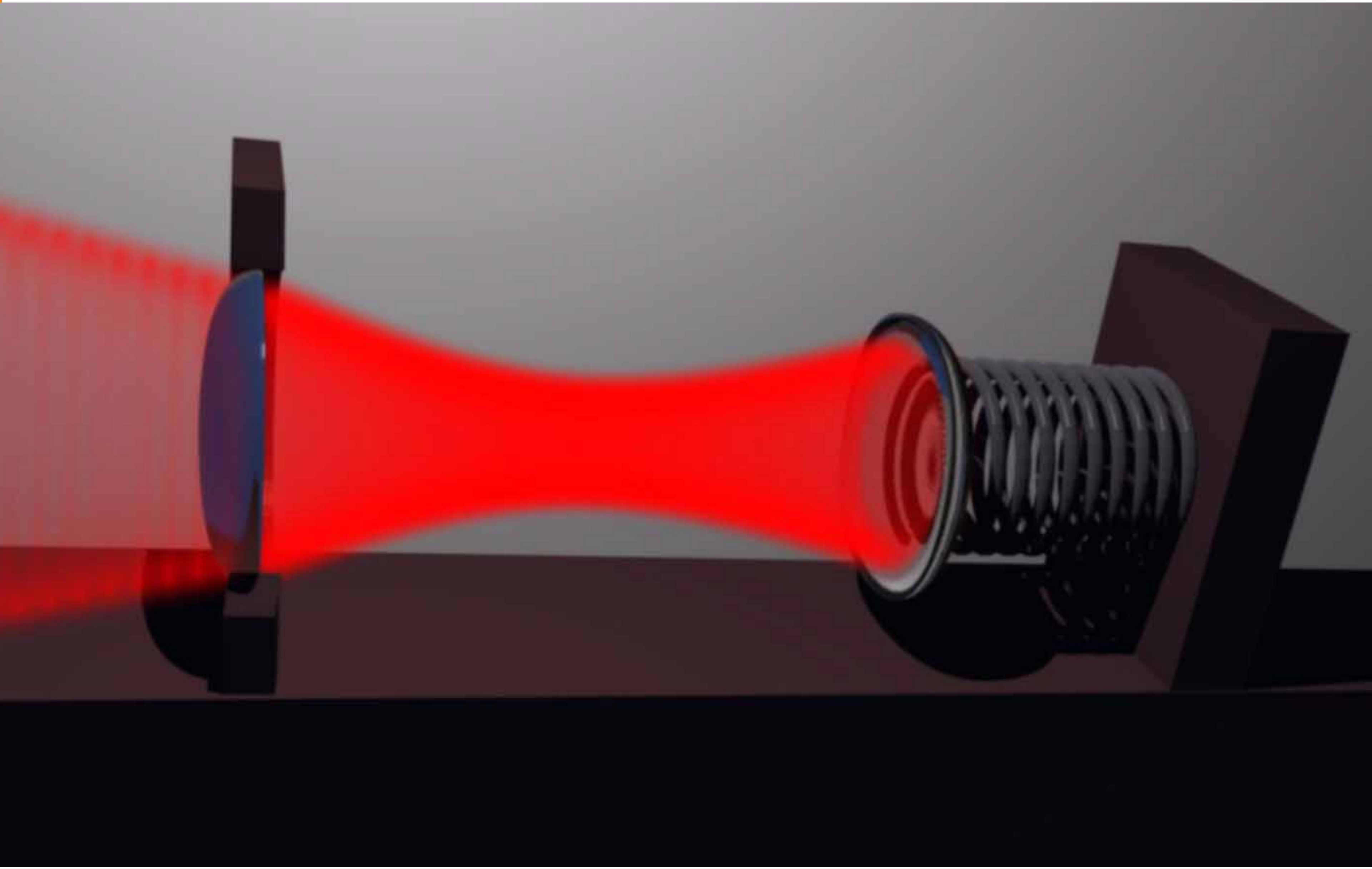
ARC CENTRE OF EXCELLENCE FOR ENGINEERED QUANTUM SYSTEMS

THE UNIVERSITY OF QUEENSLAND AUSTRALIA

Australian Government Australian Research Council

Queensland Quantum optics


QUANTUM
measurement
VLADIMIR B. BRAGINSKY
and
FARID YA. KHALILI
EDITED BY
KIP S. THORNE



[Image: Albert Schliessera,Tobias J. Kippenberg, Advances In Atomic, Molecular, and Optical Physics
Volume 58, 2010, Pages 207–323]

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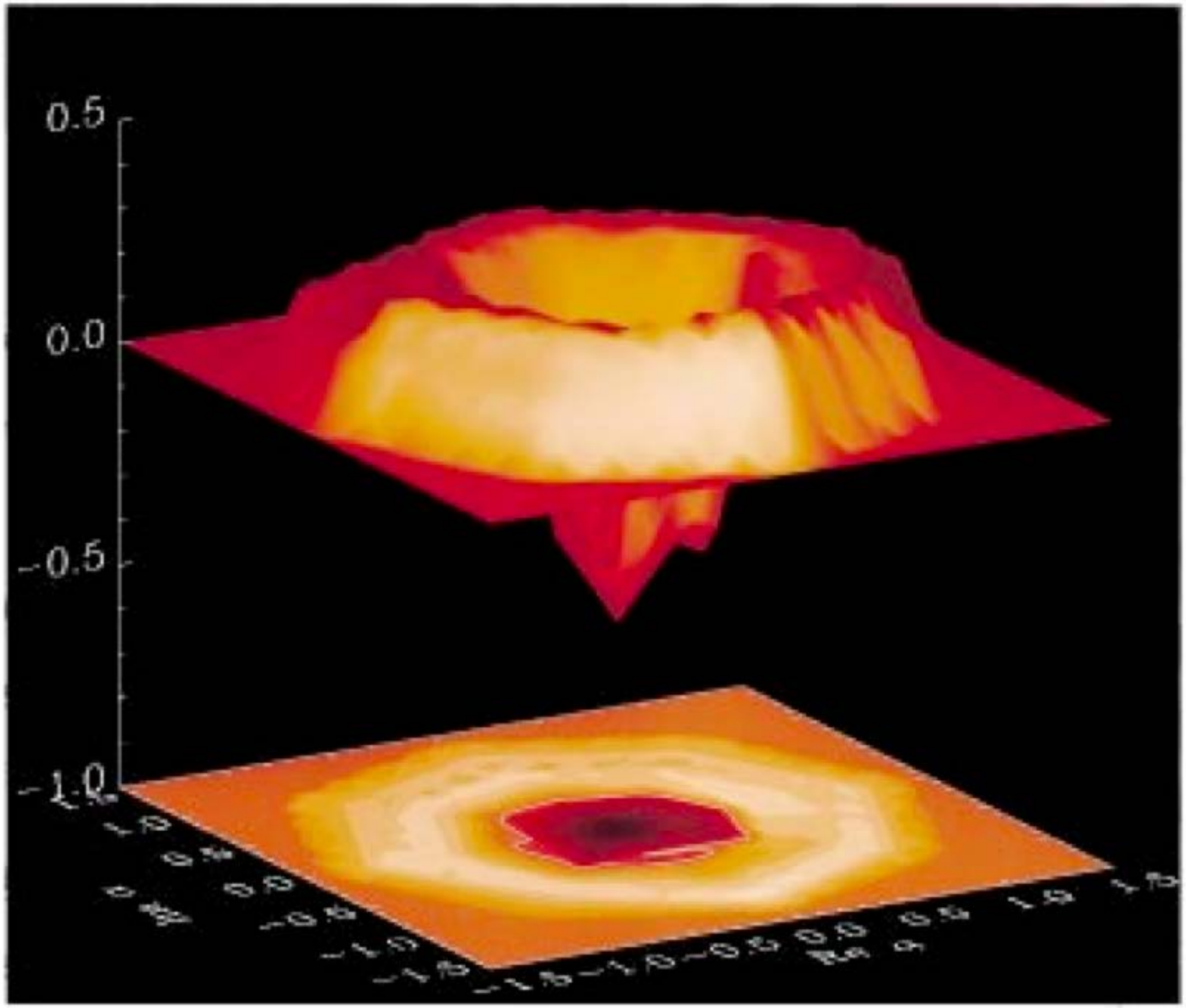


PHYSICAL REVIEW LETTERS

VOLUME 77 | 18 NOVEMBER 1996 | NUMBER 21

Experimental Determination of the Motional Quantum State of a Trapped Atom

D. Leibfried, D. M. Meekhof, B. E. King, C. Monroe, W. M. Itano, and D. J. Wineland

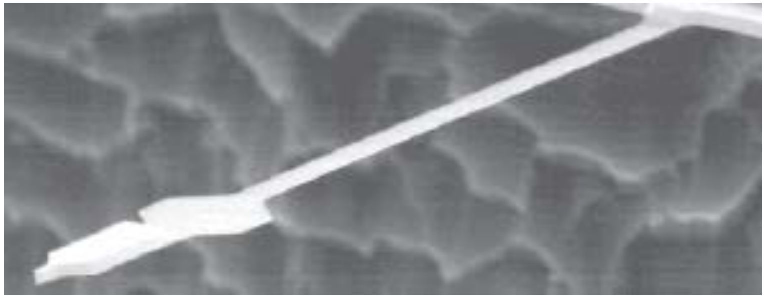
Time and Frequency Division, National Institute of Standards and Technology, Boulder, Colorado 80303-3328

(Received 11 July 1996)

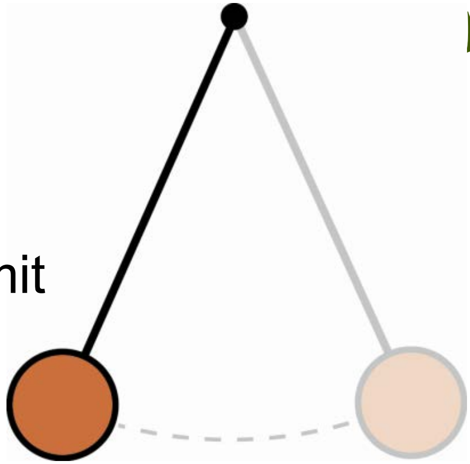
We reconstruct the density matrices and Wigner functions for various quantum states of motion of a harmonically bound $^9Be^+$ ion. We apply coherent displacements of different amplitudes and phases to the input state and measure the number state populations. Using novel reconstruction schemes we independently determine both the density matrix in the number state basis and the Wigner function. These reconstructions are sensitive indicators of decoherence in the system. [S0031-9007(96)01713-9]

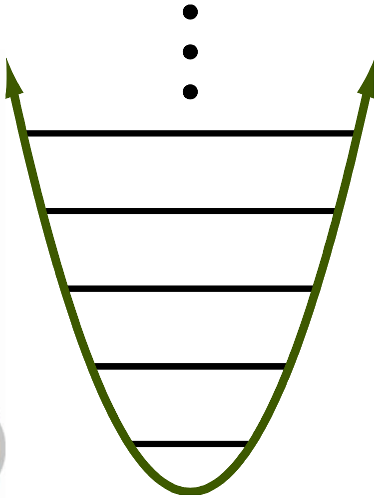
Mesoscopic quantum mechanical systems

- *Question:* Can similar experiments be performed with "large" mechanical oscillators? (~10^{15} atoms)

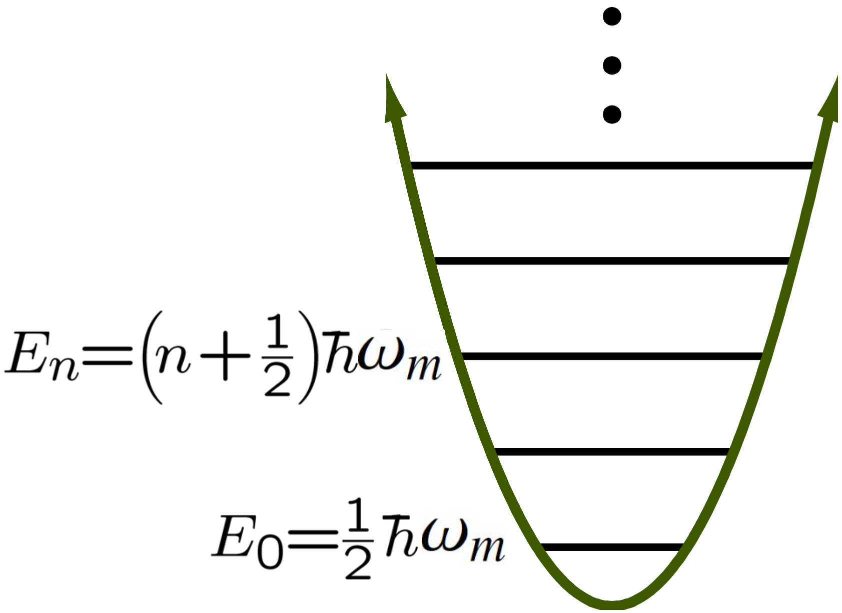


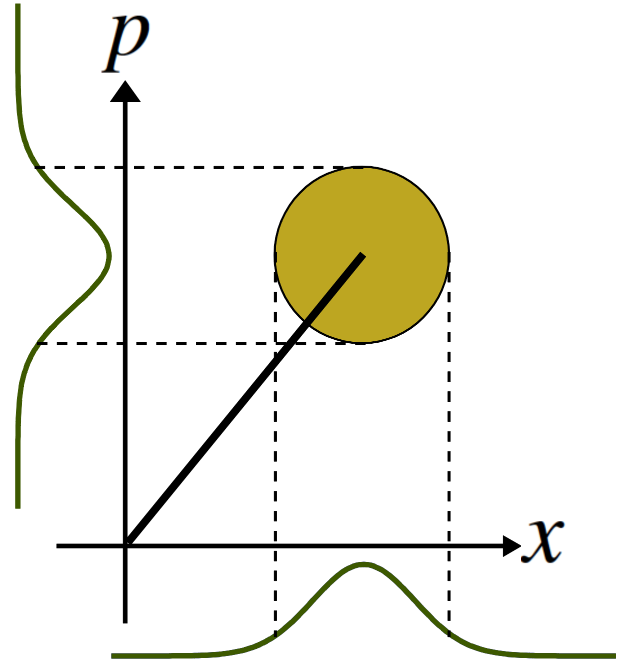
- Why?
  - Tests of fundamental physics
  - Quantum nonlinear mechanics
  - Ultra-sensitive mass/spin/force/ displacement sensors.
  - Surpass the standard quantum limit of interferometers.
  - Technology for quantum information systems…





Quantum harmonic oscillator





- Annihilation operator: $\hat{b} = \frac{1}{\sqrt{2\hbar m\omega_m}}(m\omega_m\hat{x} + i\hat{p})$

$$\hat{x} = x_{zp}\left(\hat{b} + \hat{b}^\dagger\right)$$
$$\hat{p} = ip_{zp}\left(\hat{b}^\dagger - \hat{b}\right)$$

$$[b, b^\dagger] = 1 \Rightarrow \sqrt{\langle\hat{x}^2\rangle\langle\hat{p}^2\rangle} \geq \frac{\hbar}{2}$$

Quantum *mechanics*?

SHO motion: $$\langle x^2 \rangle = \frac{\hbar}{2m\omega_m}\left\{ \underbrace{1}_{\text{ZP motion}} + \underbrace{\frac{2}{\exp\left[\frac{\hbar\omega_m}{k_B T}\right] - 1}}_{\text{Thermal motion}} \right\}$$

- Temperature: Need $T < \dfrac{\hbar\omega_m}{k_B}$ to freeze out thermal motion.

$$\omega_m \sim 10^7 \to 10^{10} \text{ rad/s} \Rightarrow \boxed{T \sim 30 \text{ mK} \to 30\ \mu\text{K}}$$

- Transduction: Need to be able to “see” ZP motion

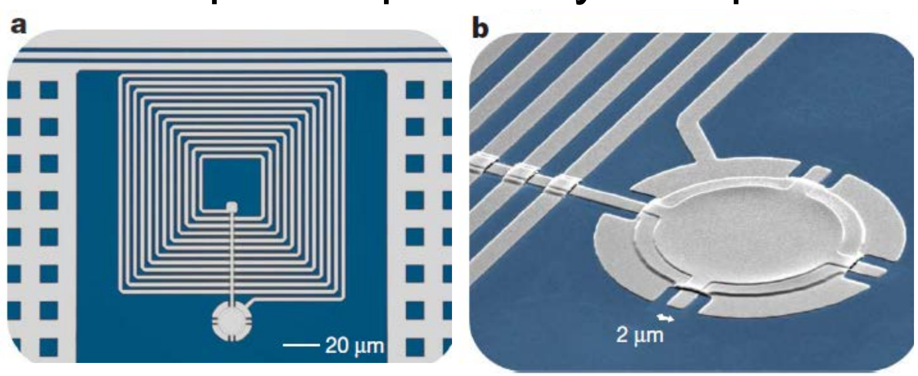
$$m = 10^{-15} \text{ kg} \Rightarrow \boxed{\langle x^2 \rangle^{1/2} = 10^{-15} \to 10^{-13} \text{ m}}$$

- Nonlinearities: Need access to nonlinearities to engineer non-classical states

Two complimentary approaches

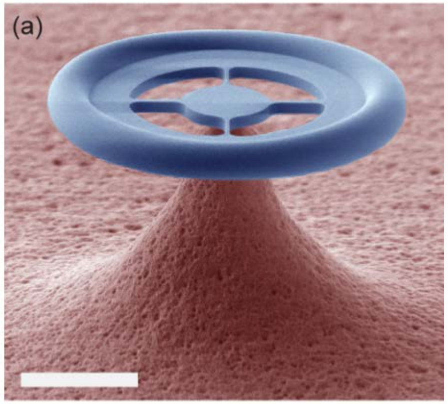
Electromechanics

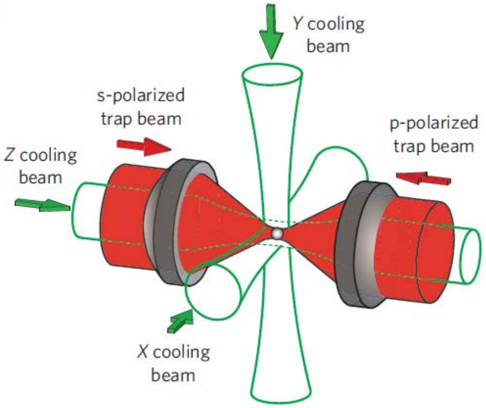
- Vibrating cantilever coupled capacitively to superconducting circuit.



Optomechanics

- Vibrating cantilever coupled via radiation pressure to optical field.



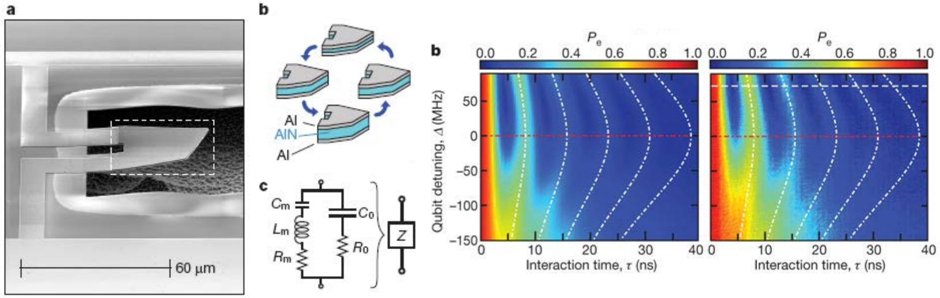


Vol 464 | 1 April 2010 | doi:10.1038/nature08967 nature

ARTICLES

Quantum ground state and single-phonon control of a mechanical resonator

A. D. O'Connell[1], M. Hofheinz[1], M. Ansmann[1], Radoslaw C. Bialczak[1], M. Lenander[1], Erik Lucero[1], M. Neeley[1], D. Sank[1], H. Wang[1], M. Weides[1], J. Wenner[1], John M. Martinis[1] & A. N. Cleland[1]

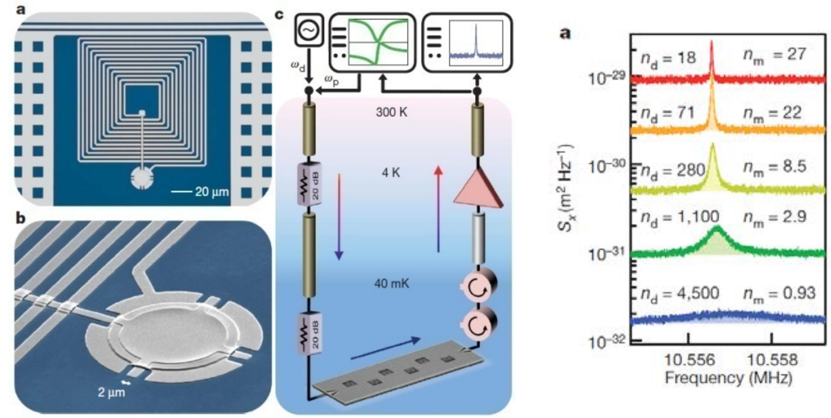


LETTER

doi:10.1038/nature10261

Sideband cooling of micromechanical motion to the quantum ground state

J. D. Teufel[1], T. Donner[2,3], Dale Li[1], J. W. Harlow[2,3], M. S. Allman[1,3], K. Cicak[1], A. J. Sirois[1,3], J. D. Whittaker[1,3], K. W. Lehnert[2,3] & R. W. Simmonds[1]

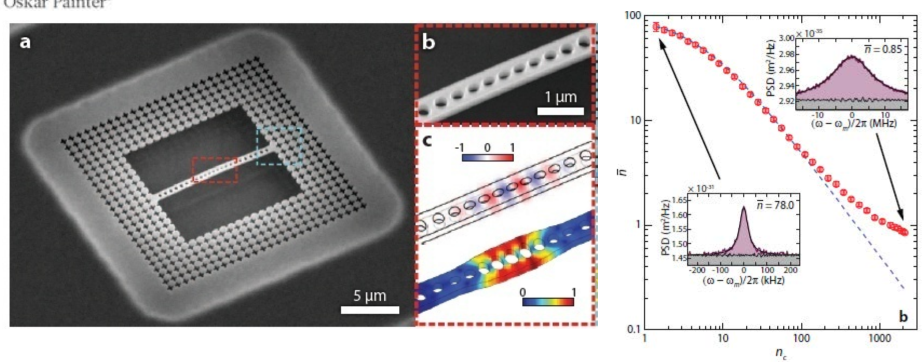


LETTER

doi:10.1038/nature10461

Laser cooling of a nanomechanical oscillator into its quantum ground state

Jasper Chan[1], T. P. Mayer Alegre[1]†, Amir H. Safavi-Naeini[1], Jeff T. Hill[1], Alex Krause[1], Simon Gröblacher[1,2], Markus Aspelmeyer[2] & Oskar Painter[1]

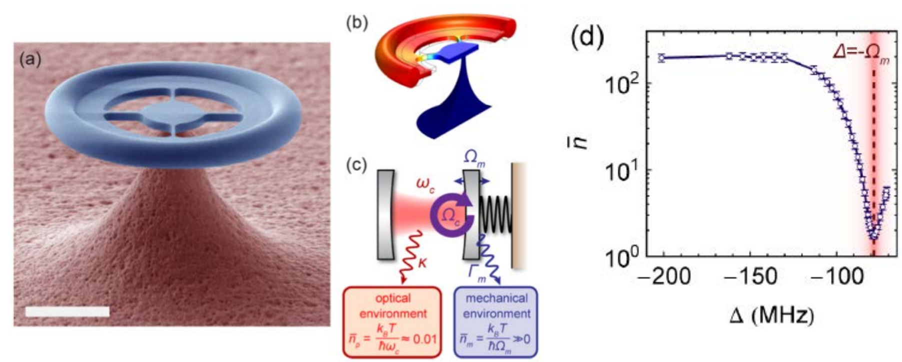


LETTER

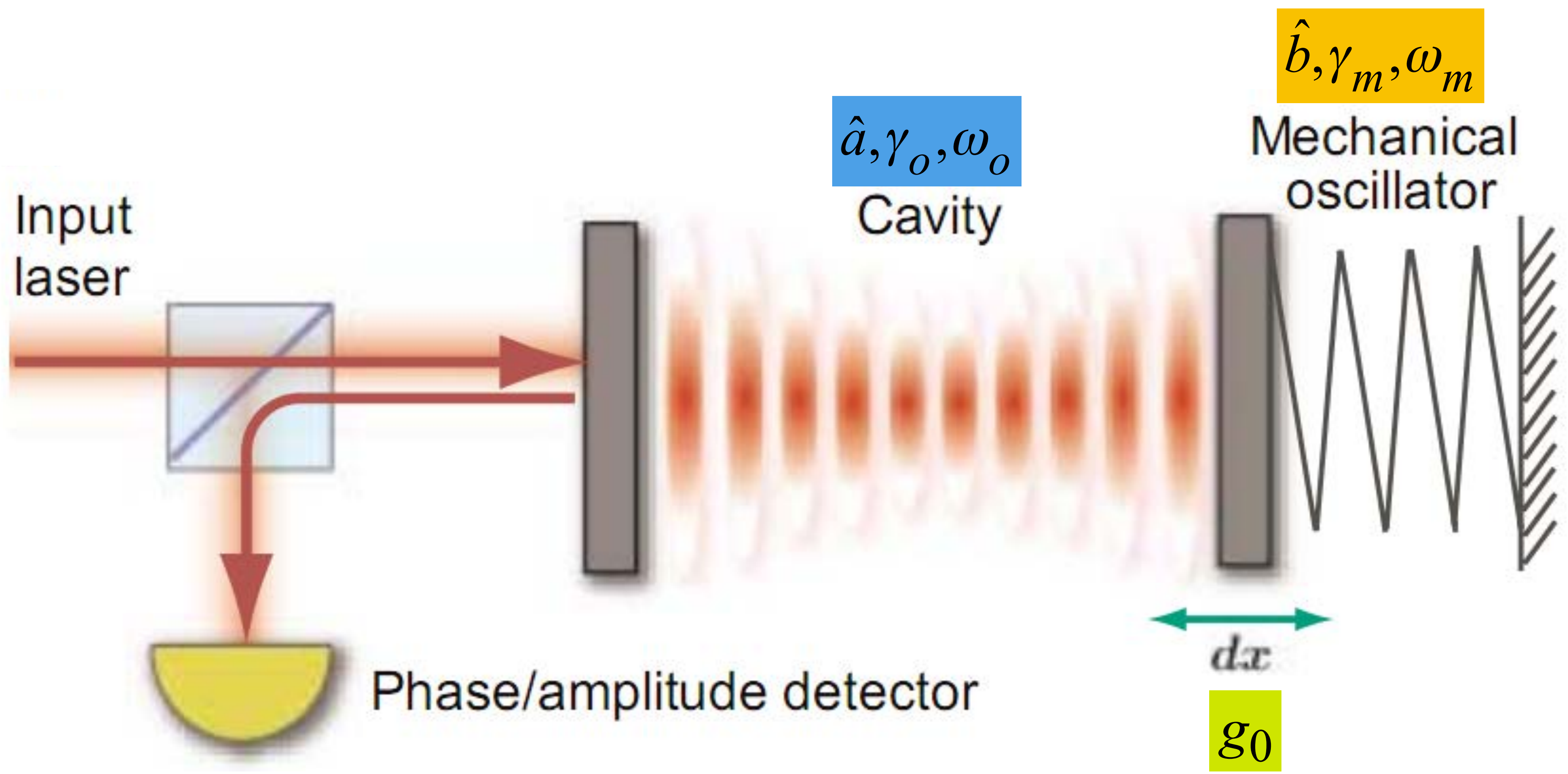
doi:10.1038/nature10787

Quantum-coherent coupling of a mechanical oscillator to an optical cavity mode

E. Verhagen[1]*, S. Deléglise[1]*, S. Weis[1,2]*, A. Schliesser[1,2]* & T. J. Kippenberg[1,2]

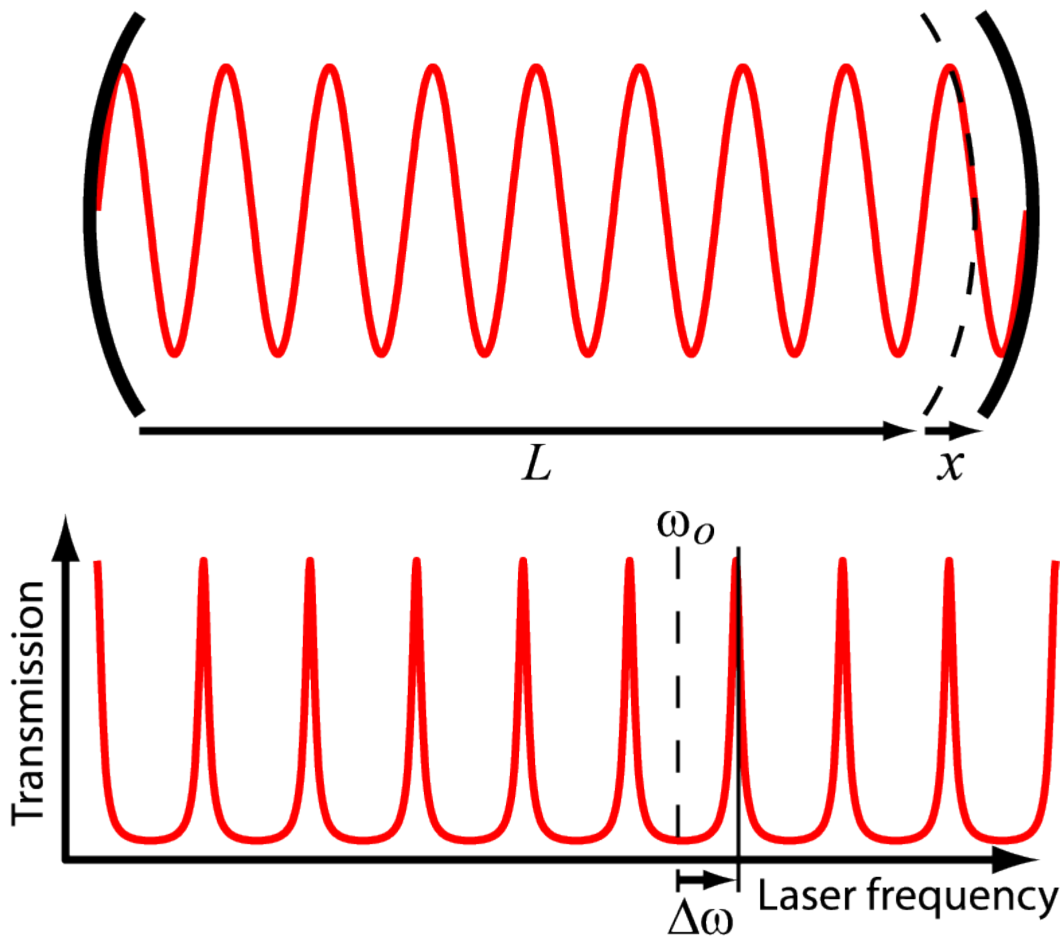


Cavity optomechanics



[Image: Kippenberg *et al*, Science **321** 1172 (2008)]

Optomechanical interaction



- Change in cavity length → shift in optical resonance frequency (and hence energy)
- For small displacement

$$\frac{\Delta\omega}{\omega_o} = \frac{x}{L}$$

$$\boxed{\frac{\Delta\omega}{x} = \frac{\omega_o}{L} = G}$$

Cavity optomechanics

$$H = \underbrace{\hbar\left(\omega_o}_{\text{Bare cavity}} + \underbrace{G\hat{x}\right)}_{\text{Optomechanical interaction}}\hat{a}^\dagger\hat{a} + \underbrace{\hbar\omega_m\hat{b}^\dagger\hat{b}}_{\text{Bare mechanical oscillator}} \qquad \hat{x} = x_{zp}\left(\hat{b} + \hat{b}^\dagger\right)$$

$$= \hbar\omega_o\hat{a}^\dagger\hat{a} + \hbar\omega_m\hat{b}^\dagger\hat{b} + \hbar G\hat{a}^\dagger\hat{a}x_{zp}\left(\hat{b}^\dagger + \hat{b}\right)$$

$$= \hbar\omega_o\hat{a}^\dagger\hat{a} + \hbar\omega_m\hat{b}^\dagger\hat{b} + \hbar g_0\hat{a}^\dagger\hat{a}\left(\hat{b}^\dagger + \hat{b}\right)$$

a/b – cavity/mechanical annihilation operators

ω_o/ω_m – optical/mechanical resonance frequencies

g_0 – vacuum optomechanical coupling rate

Cavity optomechanics

$$H = \underbrace{\hbar\omega_o \hat{a}^\dagger \hat{a}}_{\text{Bare cavity}} + \underbrace{\hbar\omega_m \hat{b}^\dagger \hat{b}}_{\text{Bare mechanical oscillator}} + \underbrace{\hbar g_0 \hat{a}^\dagger \hat{a}\left(\hat{b} + \hat{b}^\dagger\right)}_{\text{Optomechanical interaction}}$$

- Move into rotating frame at frequency of optical field by performing the transformation:

$$H \to U^\dagger H U - A \qquad U = e^{-iAt/\hbar} \qquad A = \hbar\omega_L \hat{a}^\dagger \hat{a}$$

$$\Rightarrow H = \hbar\Delta \hat{a}^\dagger \hat{a} + \hbar\omega_m \hat{b}^\dagger \hat{b} + \hbar g_0 \hat{a}^\dagger \hat{a}\left(\hat{b} + \hat{b}^\dagger\right)$$

Cavity detuning: $\Delta = \omega_o - \omega_L$

Cavity optomechanics

$$H = \underbrace{\hbar\Delta\hat{a}^\dagger\hat{a}}_{\text{Bare cavity}} + \underbrace{\hbar\omega_m\hat{b}^\dagger\hat{b}}_{\text{Bare mechanical oscillator}} + \underbrace{\hbar g_0\hat{a}^\dagger\hat{a}\left(\hat{b}+\hat{b}^\dagger\right)}_{\text{Optomechanical interaction}}$$

- Generally, g_0 very small c.f. system decay rates.
  - Boost using a bright coherent field

$$\hat{a} \to \alpha + \hat{a}$$

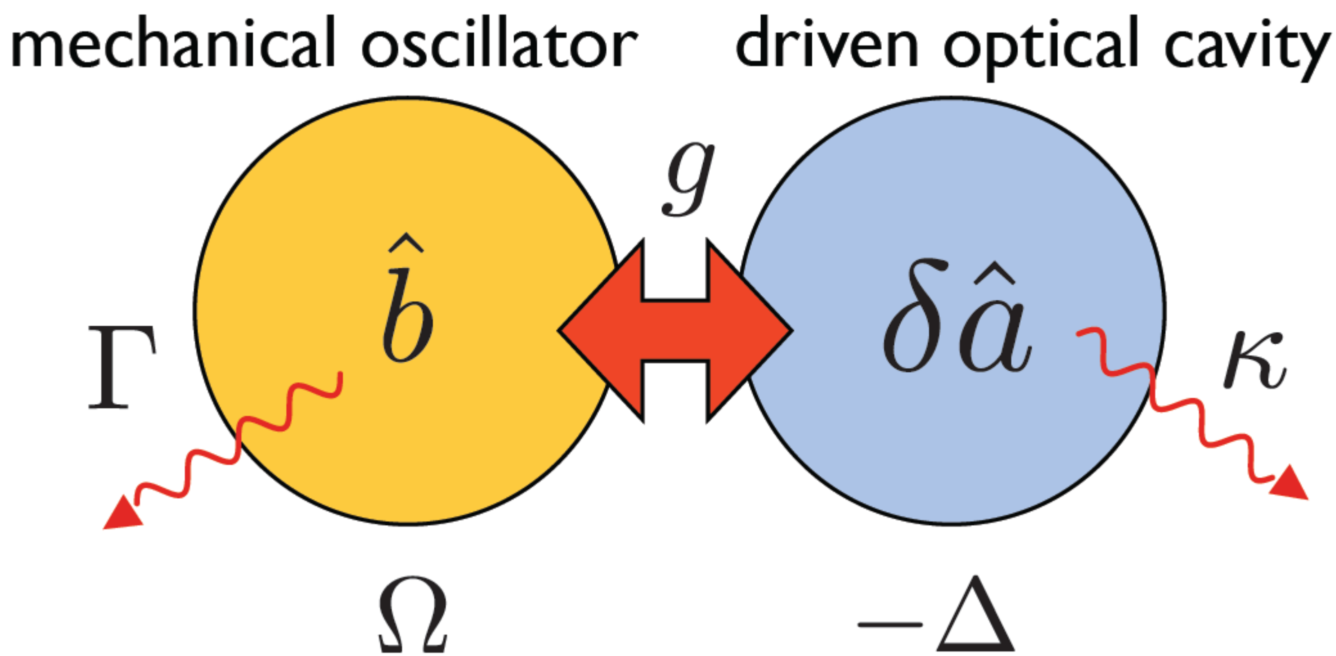
$$\hat{a}^\dagger\hat{a} \to (\alpha+\hat{a})(\alpha+\hat{a}^\dagger) = \cancel{\alpha^2} + \alpha(\hat{a}+\hat{a}^\dagger) + \cancel{\hat{a}^\dagger\hat{a}}$$

$$\hbar g_0\hat{a}^\dagger\hat{a}(\hat{b}+\hat{b}^\dagger) \to \hbar g_0\alpha(\hat{a}+\hat{a}^\dagger)(\hat{b}+\hat{b}^\dagger) = \hbar g(\hat{a}+\hat{a}^\dagger)(\hat{b}+\hat{b}^\dagger)$$

$$\boxed{H = \hbar\Delta\hat{a}^\dagger\hat{a} + \hbar\omega_m\hat{b}^\dagger\hat{b} + \hbar g\left(\hat{a}+\hat{a}^\dagger\right)\left(\hat{b}+\hat{b}^\dagger\right)}$$

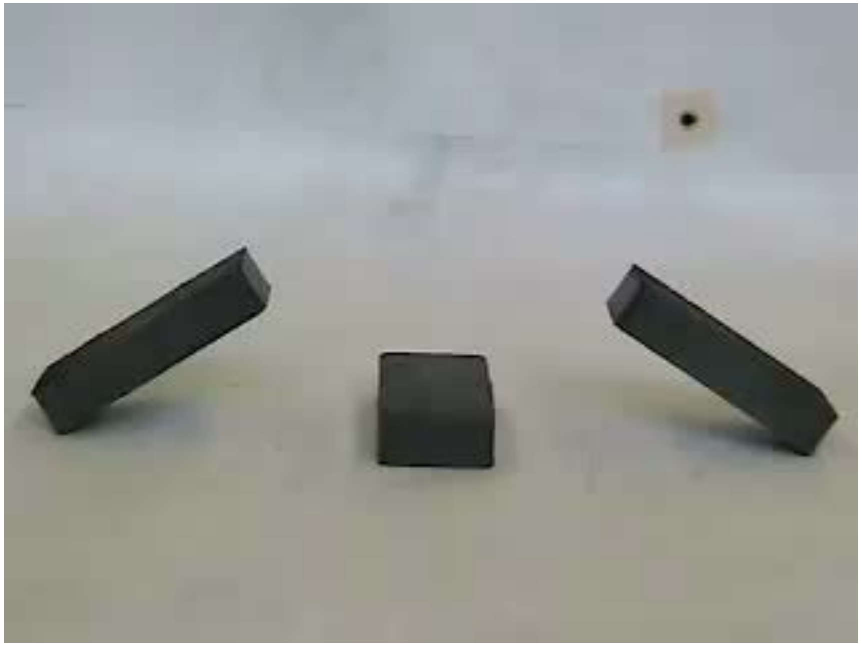
Cavity optomechanics

$$H = \hbar\Delta\hat{a}^{\dagger}\hat{a} + \hbar\omega_m \hat{b}^{\dagger}\hat{b} + \hbar g \underbrace{\left(\hat{a}+\hat{a}^{\dagger}\right)}_{X_L}\underbrace{\left(\hat{b}+\hat{b}^{\dagger}\right)}_{X_m}$$



Cavity optomechanics

$$H = \hbar\Delta\hat{a}^{\dagger}\hat{a} + \hbar\omega_m\hat{b}^{\dagger}\hat{b} + \hbar g\underbrace{\left(\hat{a} + \hat{a}^{\dagger}\right)}_{X_L}\underbrace{\left(\hat{b} + \hat{b}^{\dagger}\right)}_{X_m}$$

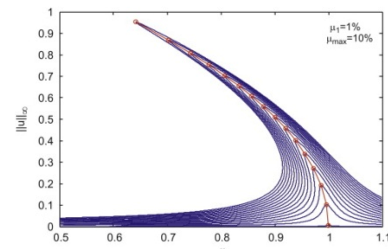


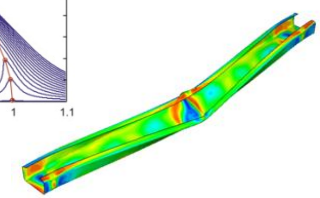
Cavity optomechanics

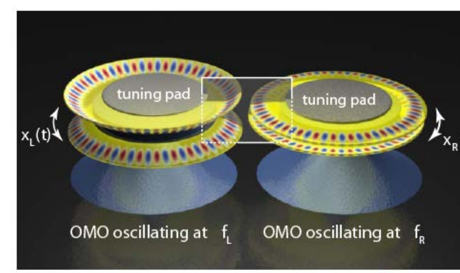
$$H = \underbrace{\hbar\Delta\hat{a}^{\dagger}\hat{a}}_{\text{Bare cavity}} + \underbrace{\hbar\omega_m\hat{b}^{\dagger}\hat{b}}_{\text{Bare mechanical oscillator}} + \underbrace{\hbar g\left(\hat{a}+\hat{a}^{\dagger}\right)\left(\hat{b}+\hat{b}^{\dagger}\right)}_{\text{Optomechanical interaction}}$$

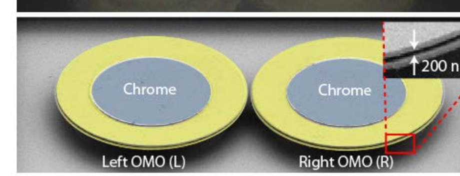
Other possible terms in Hamiltonian

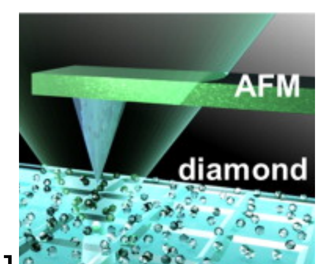
- Mechanical nonlinearities:
  - Duffing: $h g_d\left(\hat{b}+\hat{b}^{\dagger}\right)^4$
  - Parametric: $h g_p\left(\hat{b}^2+\hat{b}^{\dagger 2}\right)$
- Interactions:
  - Multiple oscillators: $\sum_{j,k} h g_{j,k}\hat{a}_j^{\dagger}\hat{a}_j\left(\hat{b}_k+\hat{b}_k^{\dagger}\right)$
  - Atomic spin: $h g_s\left(\hat{b}+\hat{b}^{\dagger}\right)S_z$
  - …











[*Review*: Milburn and Woolley *Acta Phys. Slov.* **61** 483 (2011)]

Cavity optomechanics

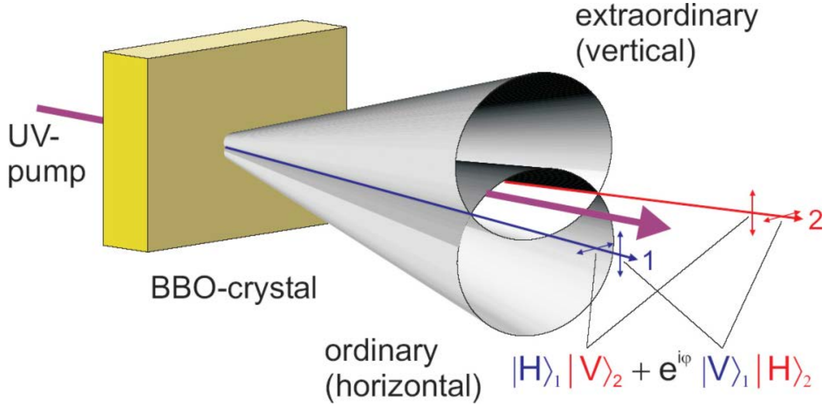
ARC CENTRE OF EXCELLENCE FOR ENGINEERED QUANTUM SYSTEMS

$$H = \hbar\Delta\hat{a}^{\dagger}\hat{a} + \hbar\omega_m\hat{b}^{\dagger}\hat{b} + \hbar g\underbrace{\left(\hat{a}+\hat{a}^{\dagger}\right)}_{X_L}\underbrace{\left(\hat{b}+\hat{b}^{\dagger}\right)}_{X_m}$$

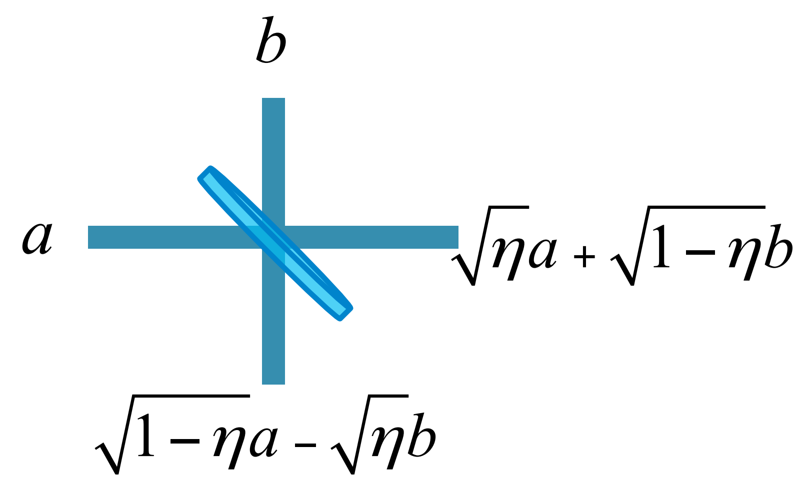
Cavity optomechanics

$$H_I = \hbar g\left(\underbrace{\hat{a}\hat{b} + \hat{a}^\dagger\hat{b}^\dagger}_{\text{Optomechanical entanglement}} + \underbrace{\hat{a}\hat{b}^\dagger + \hat{a}^\dagger\hat{b}}_{\text{Optomechanical beam splitter}}\right)$$

Analog: Parametric down conversion



Analog: Optical beam splitter



[See eg Marquart Le Houches (2011)]

Controllable interaction

- *On resonance* $\Delta = 0$

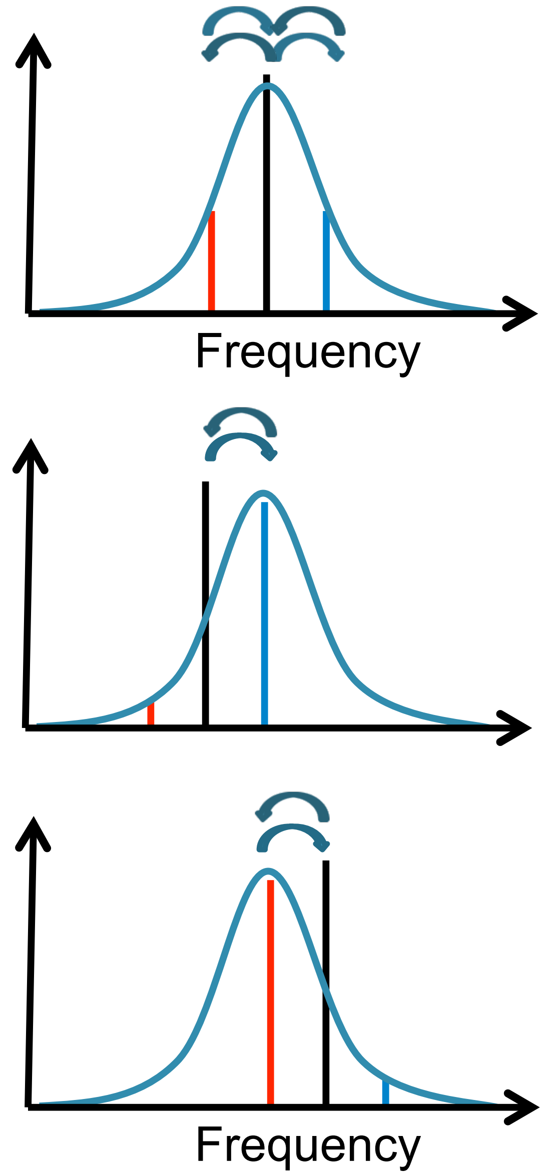
  → Readout: $H_I = \hbar g\left(\hat{a} + \hat{a}^\dagger\right)\hat{x}$

- *Red detuning* $\Delta = +\omega_m$

  → Beam splitter: $H_I \approx \hbar g\left(\hat{a}\hat{b}^\dagger + \hat{a}^\dagger\hat{b}\right)$

- *Blue detuning* $\Delta = -\omega_m$

  → Entanglement: $H_I \approx \hbar g\left(\hat{a}\hat{b} + \hat{a}^\dagger\hat{b}^\dagger\right)$



[See eg Marquart Le Houches (2011)]

Two types of optomechanical systems

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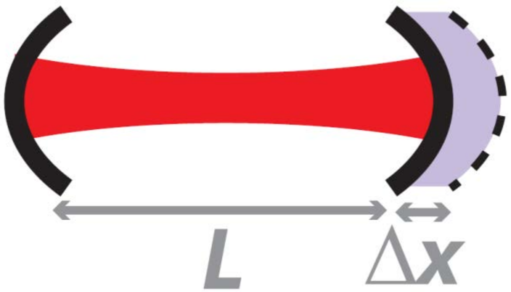
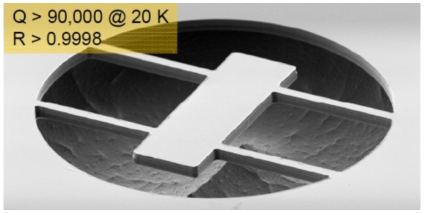
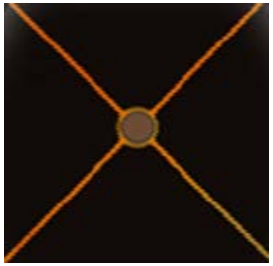
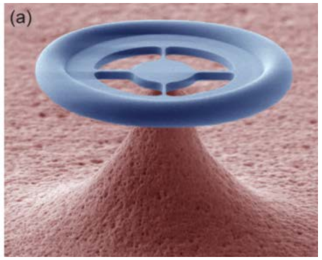
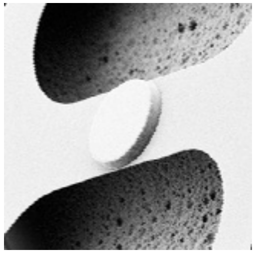
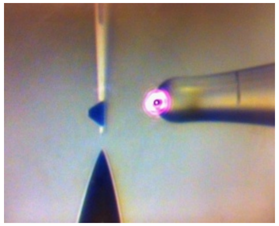


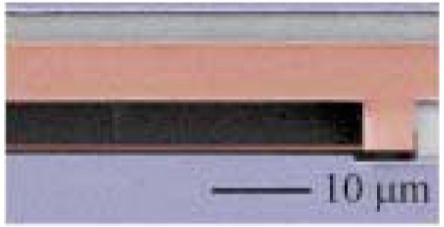


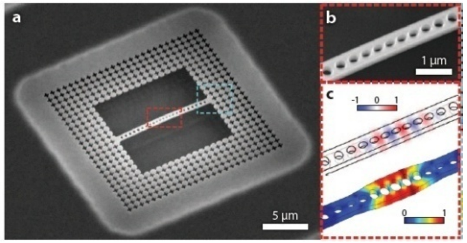


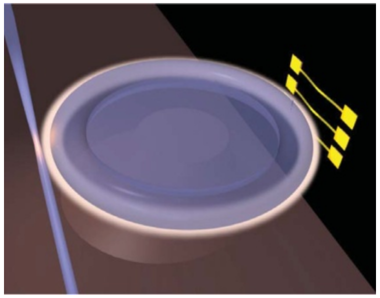












Current state-of-the-art

						Strong coupling		Cooperativity	
System	$g_0/2\pi$ [Hz]	$\kappa/2\pi$ [Hz]	$\Gamma_m/2\pi$ [Hz]	Ω_m/κ	x_{zpf} [fm]	$\frac{g_0}{\max\{\kappa,\Gamma\}}$	$\frac{g_0}{\max\{\kappa,n_{bath}\Gamma\}}$	$\frac{g_0}{\sqrt{\kappa\Gamma}}$	$\frac{g_0}{\sqrt{\kappa n_{bath}\Gamma}}$
Crystalline microresonator	1	2×10^5	1×10^3	12	0.003	10^{-6}	10^{-7}	10^{-5}	10^{-7}
Movable mirror	2×10^{-3}	2×10^4	0.06	0.01	0.01	10^{-8}	10^{-9}	10^{-5}	10^{-9}
Microsphere	1×10^3	3×10^7	8×10^4	5	0.04	10^{-5}	10^{-5}	10^{-4}	10^{-5}
Micromirror	1	2×10^6	80	0.4	0.01	10^{-7}	10^{-7}	10^{-5}	10^{-7}
Micromirror	5	8×10^5	30	1	0.5	10^{-6}	10^{-6}	10^{-4}	10^{-6}
Micromirror	300	8×10^8	600	0.001	40	10^{-7}	10^{-7}	10^{-4}	10^{-6}
Spoke-microresonator	500	5×10^6	500	5	0.2	10^{-5}	10^{-5}	10^{-3}	10^{-4}
Membrane-in-the-middle	5	2×10^5	0.1	0.6	1	10^{-6}	10^{-6}	10^{-2}	10^{-5}
Double-microdisk	8×10^4	1×10^8	2×10^3	0.06	3	10^{-4}	10^{-4}	10^{-1}	10^{-3}
Optomechanical crystal	2×10^5	5×10^9	2×10^6	0.5	3	10^{-5}	10^{-5}	10^{-3}	10^{-4}
Photonic crystal cavity	6×10^5	2×10^9	8×10^4	0.004	5	10^{-4}	10^{-4}	10^{-2}	10^{-4}
Nanomechanical rod	–	8×10^8	300	0.002	–	-	-	-	-
Near-field nanomechanics	500	5×10^6	100	2	20	10^{-5}	10^{-5}	10^{-2}	10^{-4}
Near-field nanomechanics	50	2×10^9	5×10^4	0.02	20	10^{-8}	10^{-8}	10^{-6}	10^{-7}
Microwave nanomechanics*	1	3×10^6	10	0.4	30	10^{-7}	10^{-7}	10^{-4}	10^{-5}
Microwave nanomechanics*	2	6×10^5	6	2	30	10^{-6}	10^{-6}	10^{-3}	10^{-5}

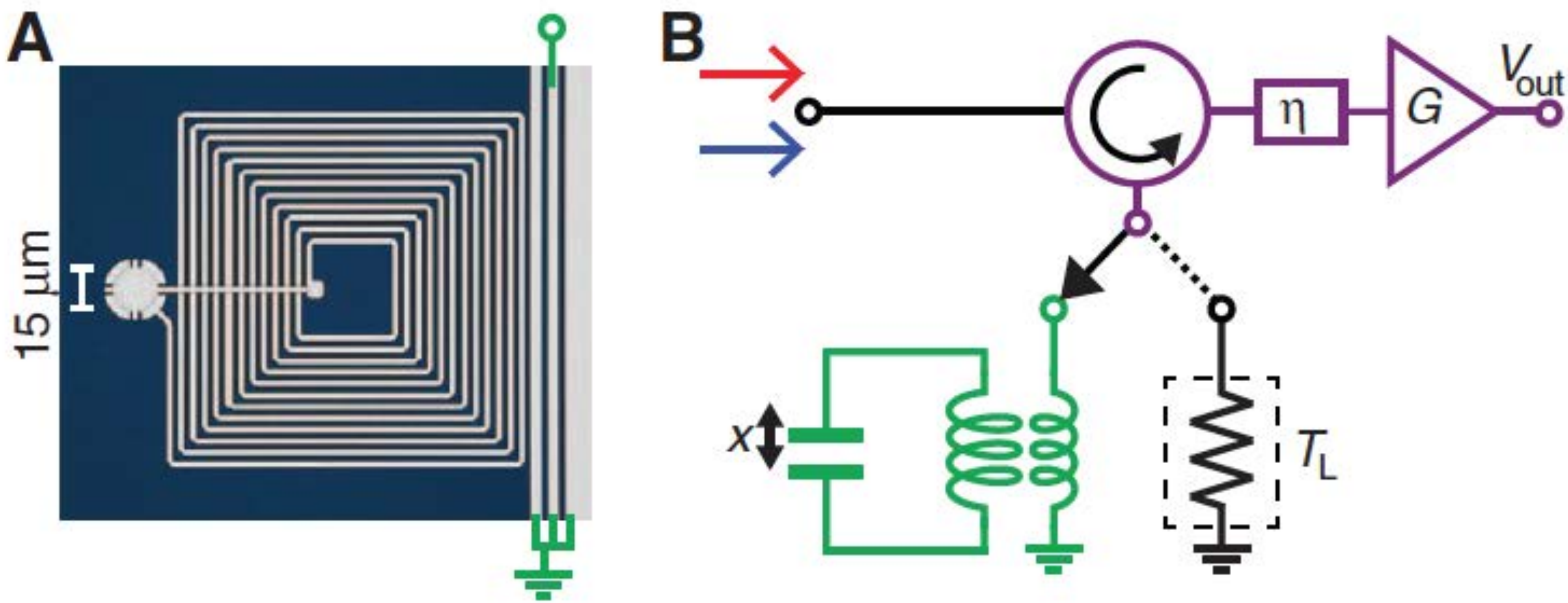
Classical

Quantum enabled

[Anetsberger et al, C. R. Physique **12** 800 (2011)]

Recent results

Entangling Mechanical Motion with Microwave Fields
T. A. Palomaki *et al.*
Science **342**, 710 (2013);
DOI: 10.1126/science.1244563



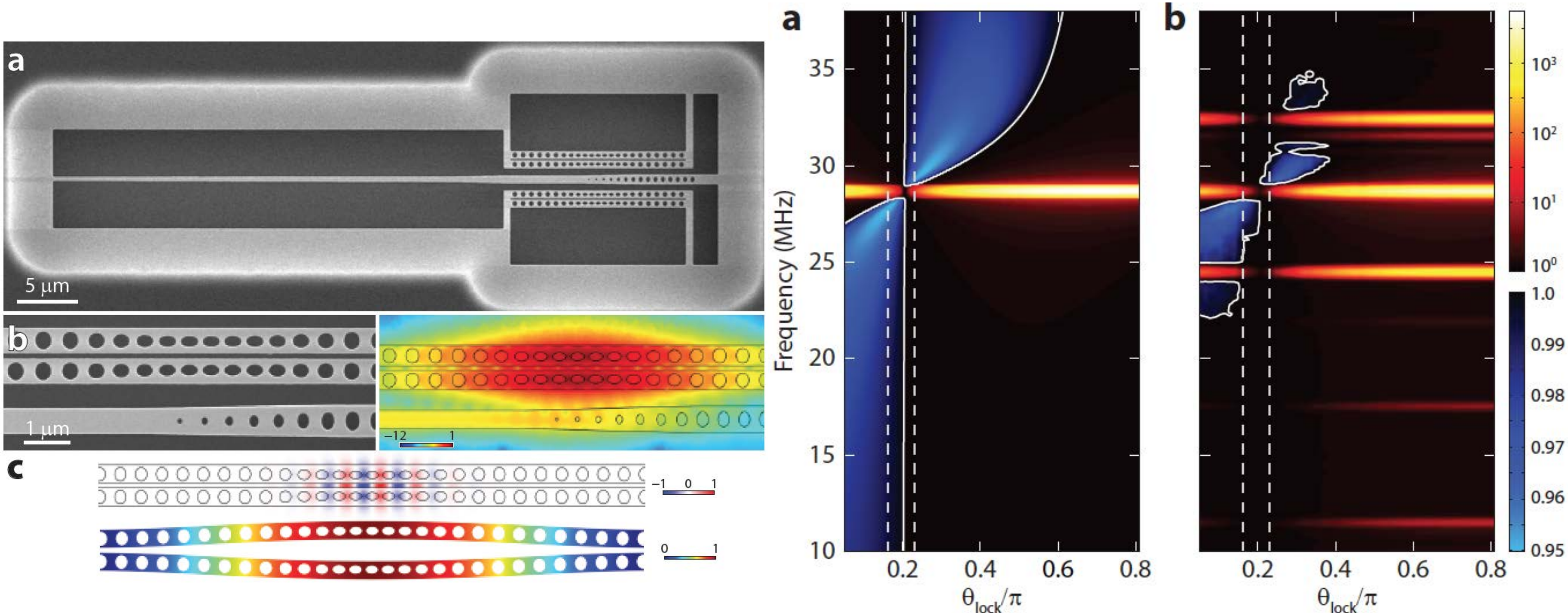
Recent results

LETTER

doi:10.1038/nature12307

Squeezed light from a silicon micromechanical resonator

Amir H. Safavi-Naeini[1,2*], Simon Gröblacher[1,2*], Jeff T. Hill[1,2*], Jasper Chan[1], Markus Aspelmeyer[3] & Oskar Painter[1,2,4]

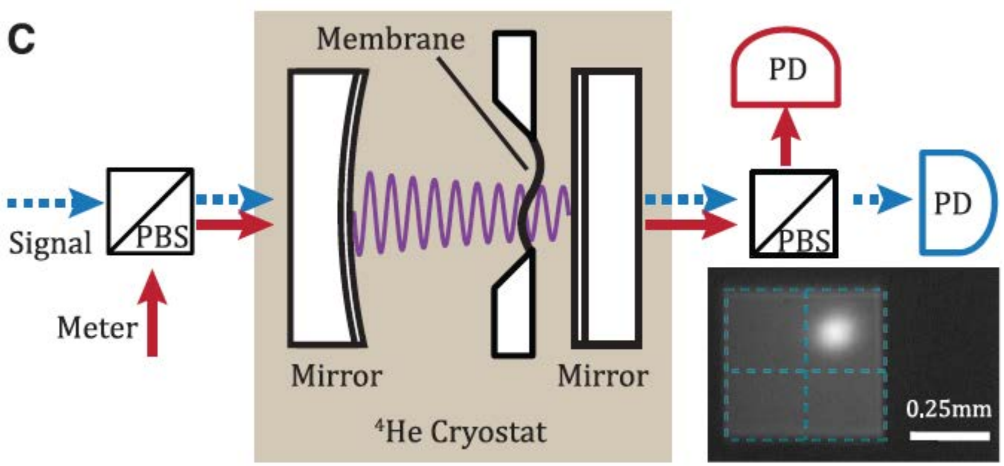


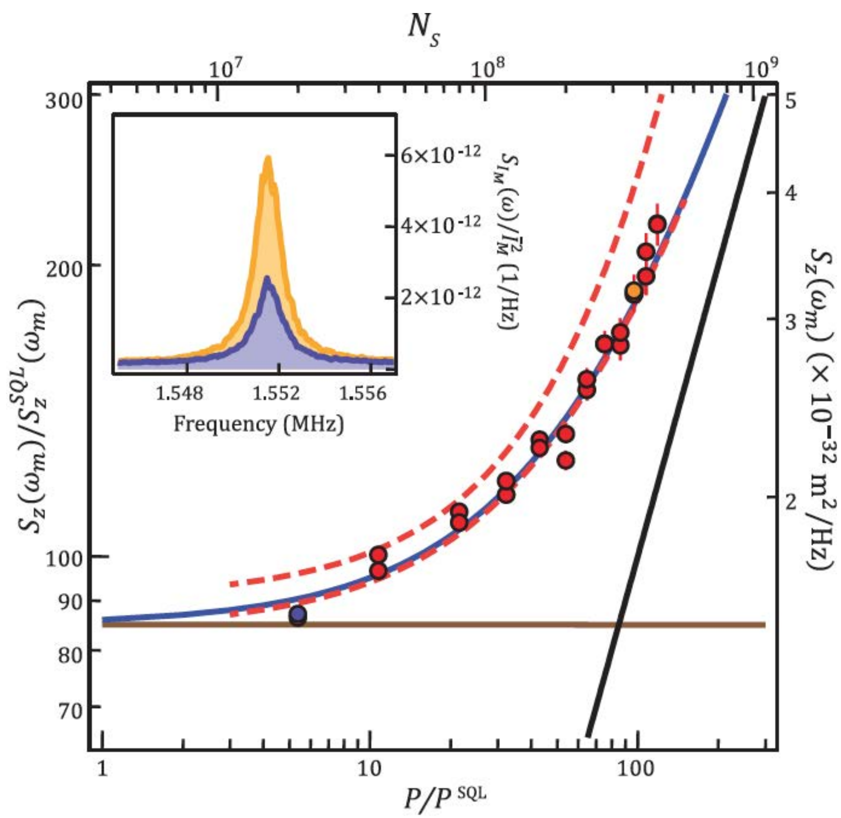
Recent results

Science AAAS

Observation of Radiation Pressure Shot Noise on a Macroscopic Object

T. P. Purdy,[1,2]* R. W. Peterson,[1,2] C. A. Regal[1,2]





Recent results

ARC CENTRE OF EXCELLENCE FOR ENGINEERED QUANTUM SYSTEMS

LETTERS
PUBLISHED ONLINE: 11 SEPTEMBER 2011 | DOI: 10.1038/NPHYS2083

nature physics

A gravitational wave observatory operating beyond the quantum shot-noise limit

The LIGO Scientific Collaboration †*

Recent results

ARC CENTRE OF EXCELLENCE FOR ENGINEERED QUANTUM SYSTEMS

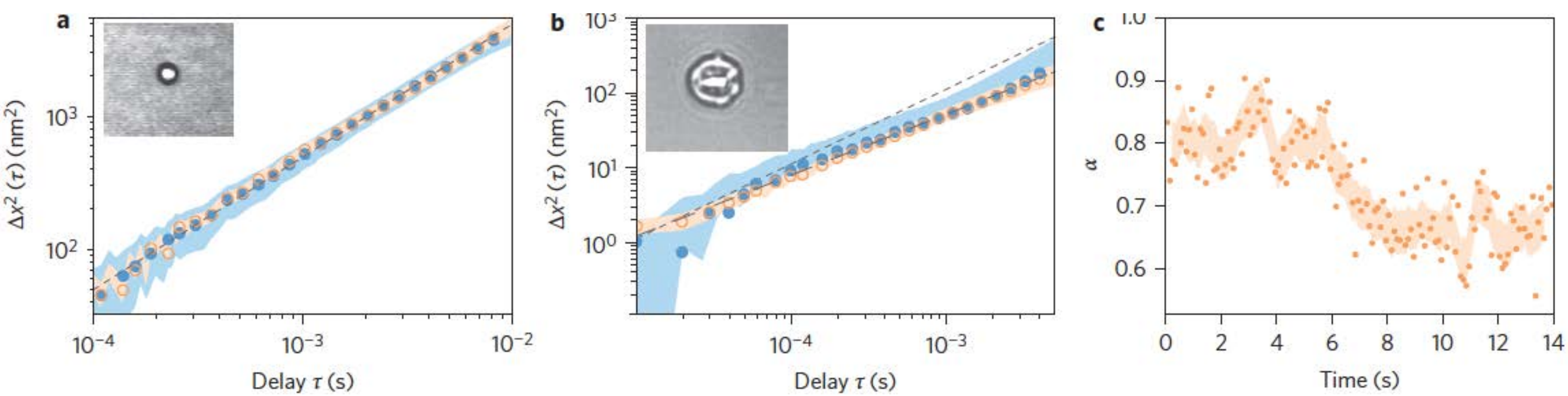
nature photonics

ARTICLES

PUBLISHED ONLINE: 3 FEBRUARY 2013 | DOI: 10.1038/NPHOTON.2012.346

Biological measurement beyond the quantum limit

Michael A. Taylor[1,2], Jiri Janousek[3], Vincent Daria[3], Joachim Knittel[1], Boris Hage[3], Hans-A. Bachor[3] and Warwick P. Bowen[2]*



Modeling cavity optomechanics

- Can model cavity optomechanical systems using
  - Unitary evolution in Schrodinger/Heisenberg pictures
  - Stochastic master equation
  - Quantum trajectories
  - Quantum Langevin equation
- Quantum Langevin equation (QLE) provides an intuitive approach (for experimentalists!).
- Heisenberg picture >>> operators evolve.
- Here, use QLE to model resolved sideband cooling and optomechanically induced transparency.

Quantum Langevin equation

- Heisenberg equation of motion:

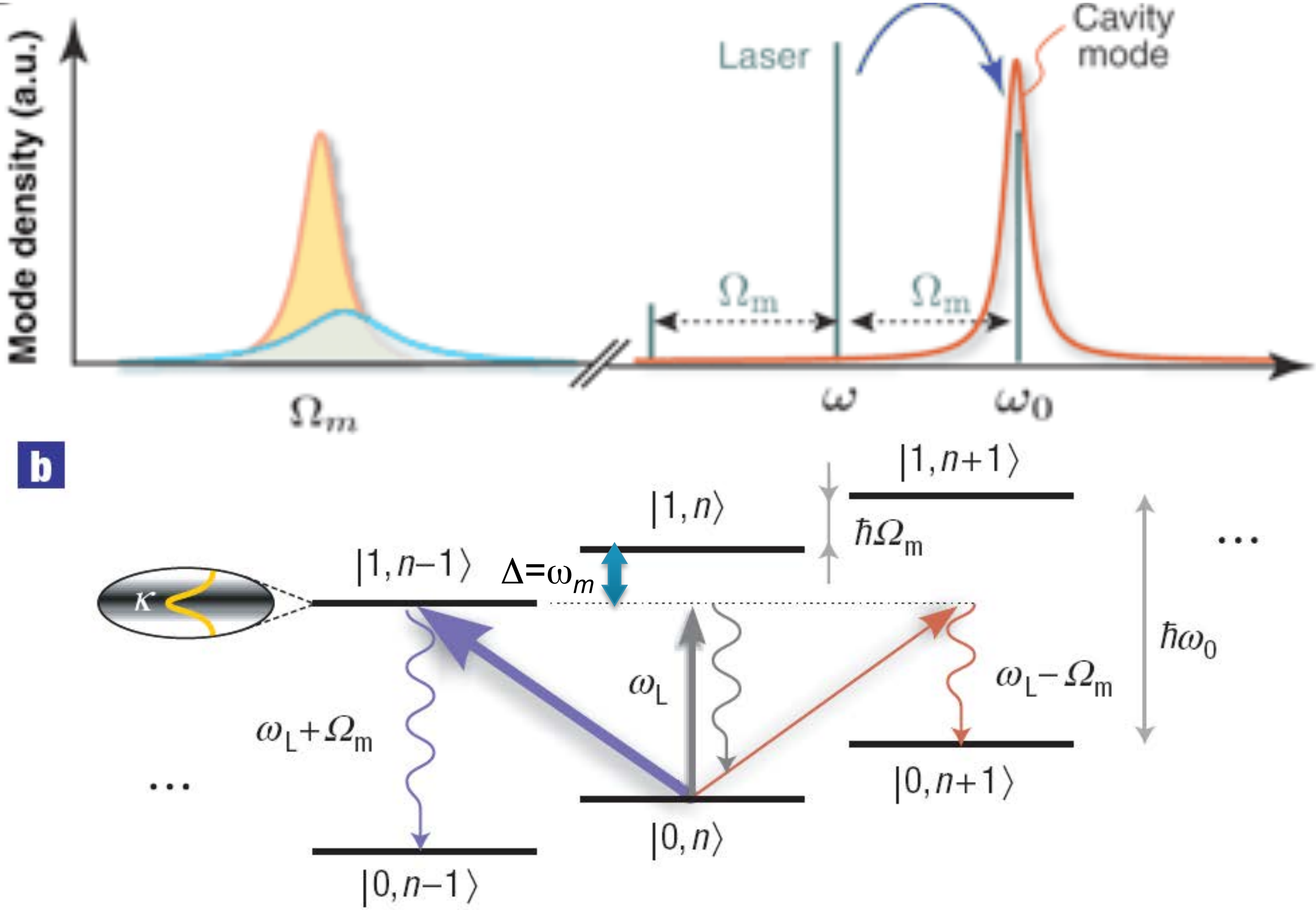
$$\dot{O} = -\frac{i}{\hbar}\left[O, \tilde{H}\right]$$

- Include mechanical and optical dissipation into Heisenberg equation to study realistic scenarios (open systems).

  ➔ Quantum Langevin equation:

$$\dot{O} = -\frac{i}{\hbar}\left[O, \tilde{H}\right] - \overbrace{\gamma O}^{\text{Dissipation}} + \underbrace{\sqrt{2\gamma}O_{\mathrm{in}}}_{\text{Fluctuation}}$$

Resolved sideband cooling



[A. Schliesser Nat. Phys. **4** 415 (2008)]

Resolved sideband cooling

$$\tilde{H} = \hbar\omega_m \hat{a}^\dagger \hat{a} + \hbar\omega_m \hat{b}^\dagger \hat{b} + \hbar g \left(\hat{a}^\dagger + \hat{a}\right)\left(\hat{b}^\dagger + \hat{b}\right)$$

- Consider "*good cavity limit*", restricting ω_m to be much greater than all other rates including the optical decay rate.
- Cooling in this regime termed "*resolved sideband cooling*".
- Can then apply rotating wave approximation, neglecting fast rotating terms

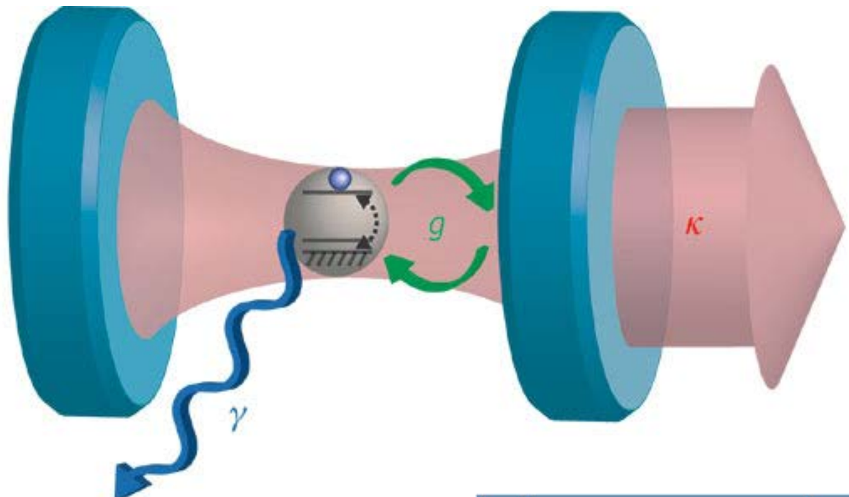
$$\tilde{H} = \hbar\omega_m \hat{a}^\dagger \hat{a} + \hbar\omega_m \hat{b}^\dagger \hat{b} + \hbar g \left(\hat{a}^\dagger \hat{b} + \hat{a}\hat{b}^\dagger\right)$$

- Fast rotating terms lead to mechanical heating and squeezing.

Challenge exercise: model resolved sideband cooling without the RWA

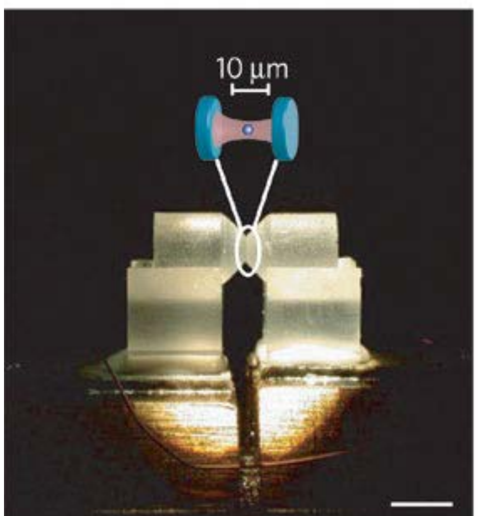
Strong coupling in cQED

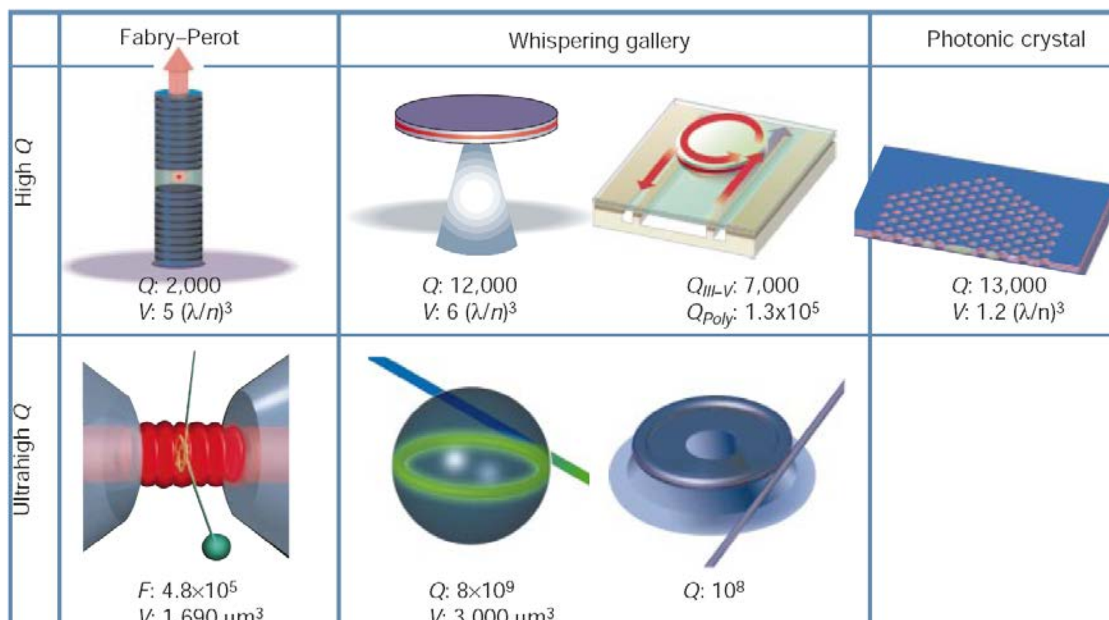
a



$$\boxed{H_I = \hbar g_0\left(\hat{a}\hat{\sigma}^\dagger + \hat{a}^\dagger\hat{\sigma}\right)}$$

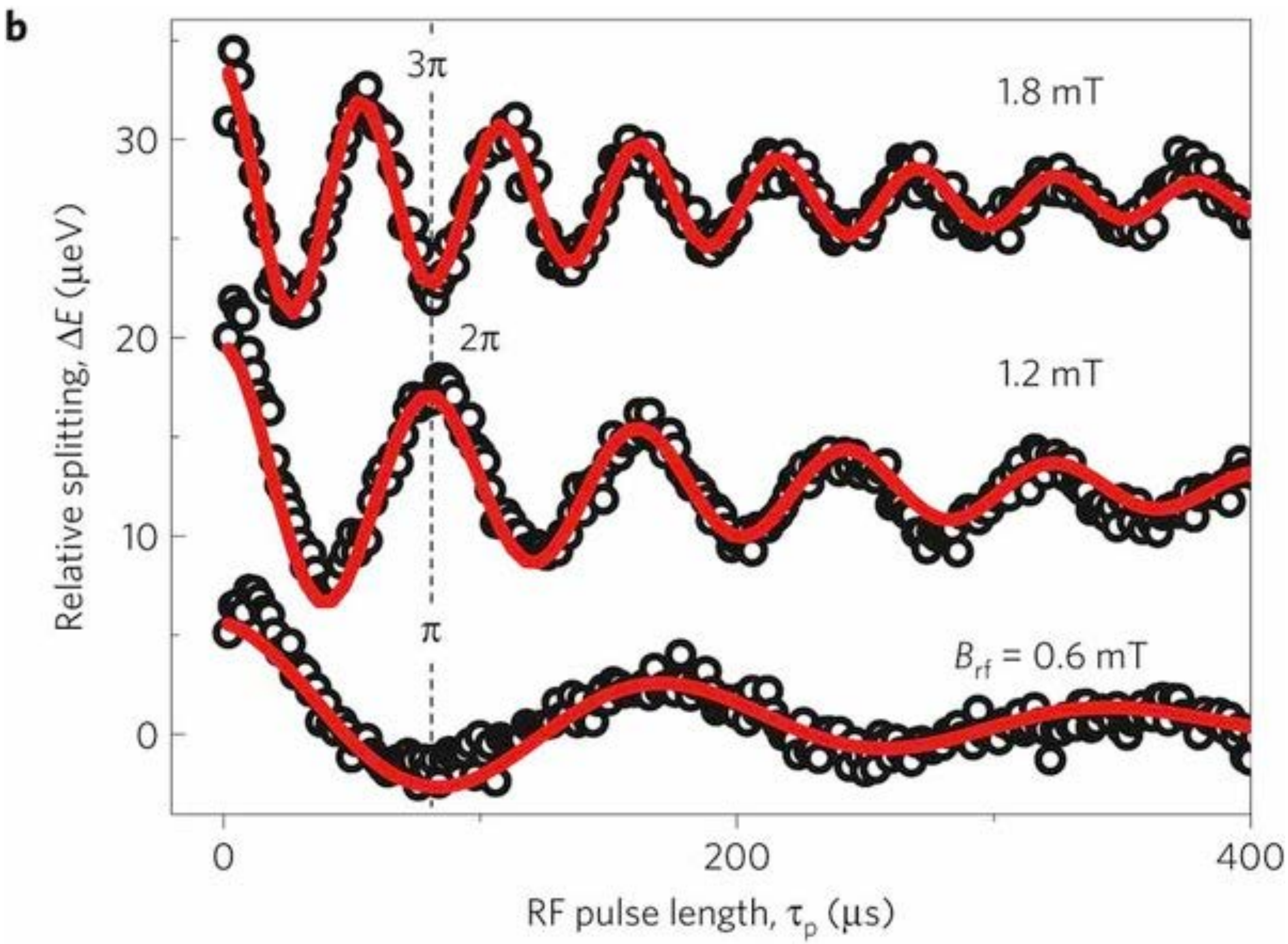
b



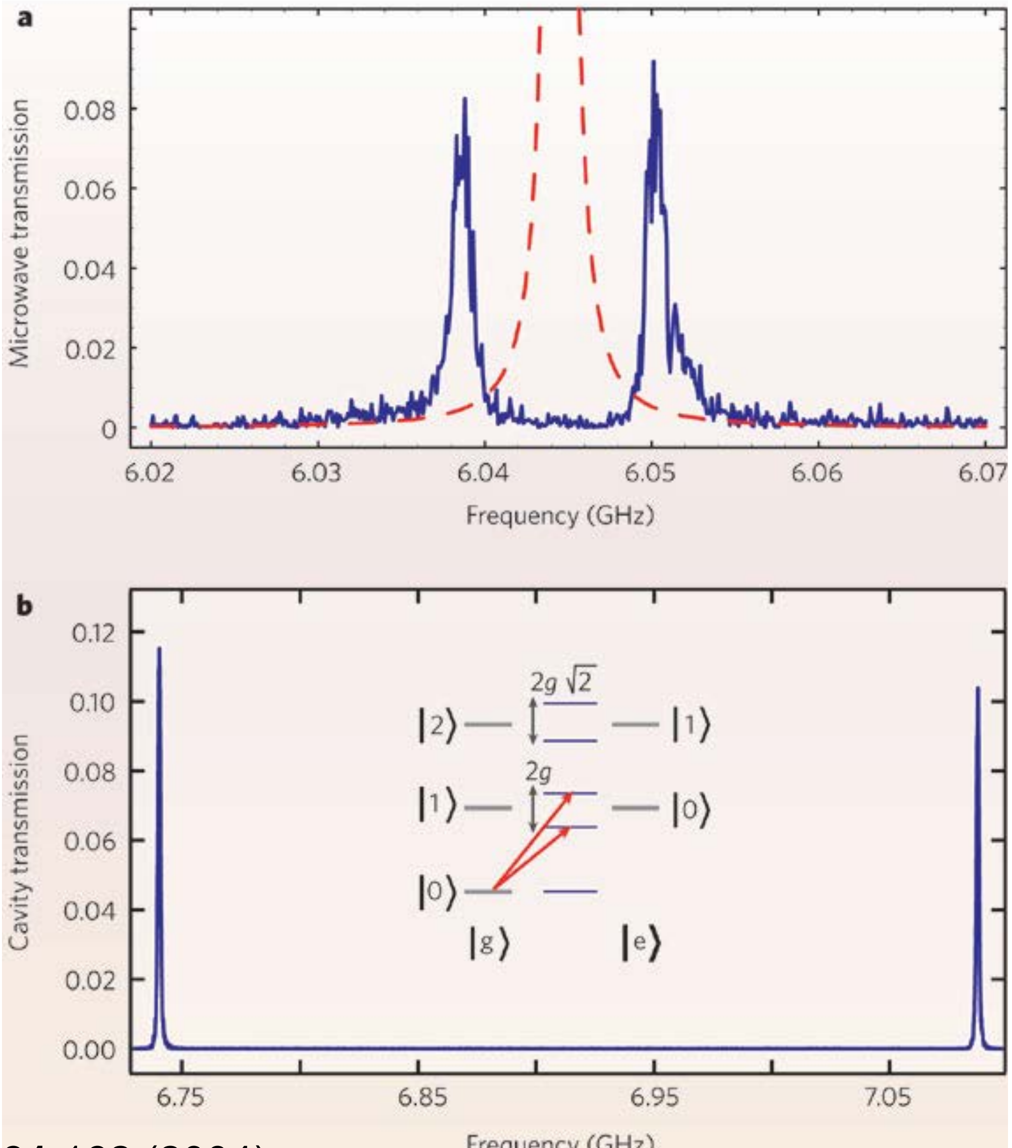


[Vahala, Nature **424** 839 (2003); Kimble, Nature **453** 1023 (2008)]

Strong coupling in cQED

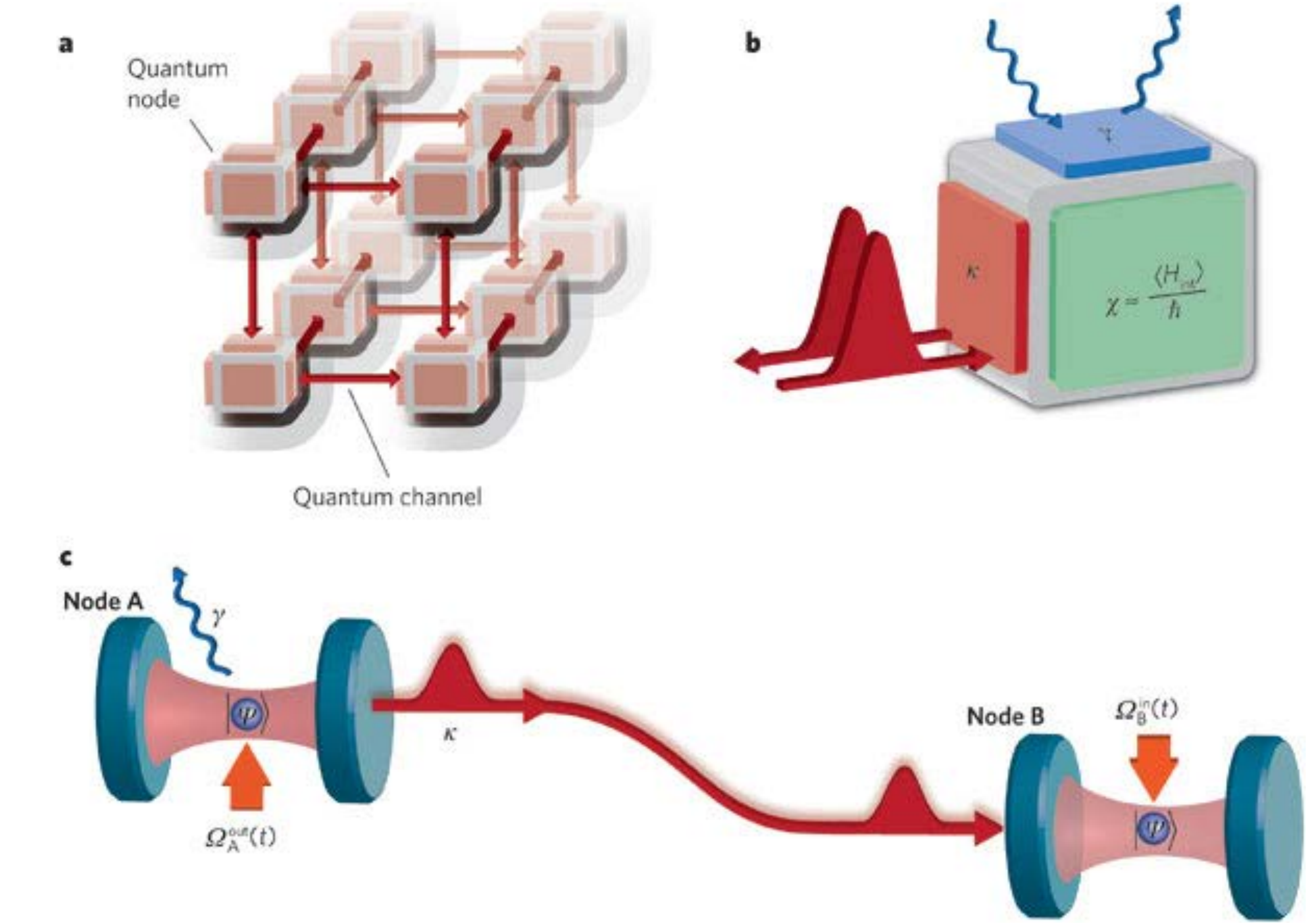


Strong coupling in cQED



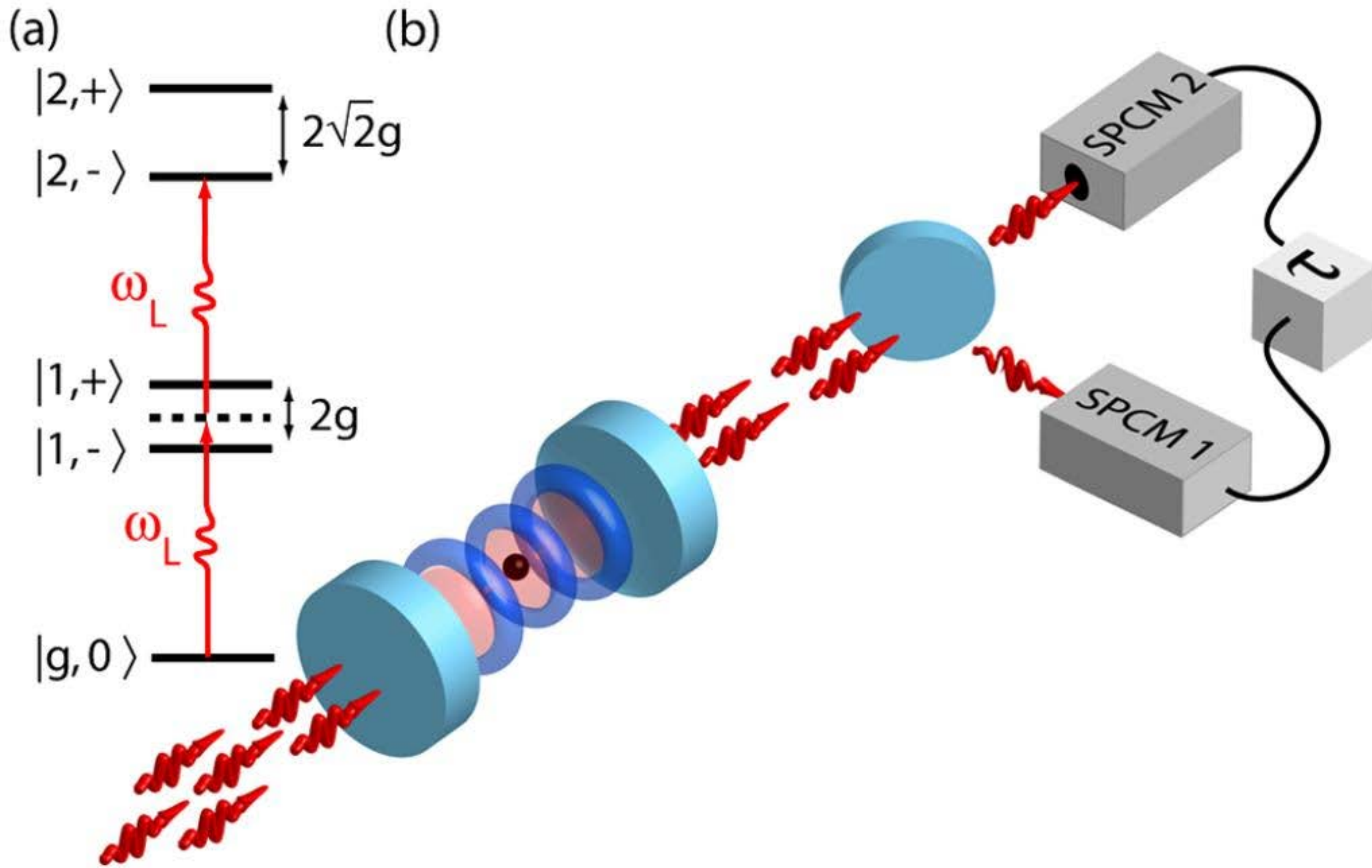
[Wallraff et al. Nature **431** 162 (2004).

Strong coupling in cQED



[Kimble, Nature **453** 1023 (2008)]

Strong coupling in cQED



[Kimble, Nature **453** 1023 (2008)]

Resolved sideband cooling

$$\tilde{H} = \hbar\omega_m \hat{a}^\dagger \hat{a} + \hbar\omega_m \hat{b}^\dagger \hat{b} + \hbar g\left(\hat{a}^\dagger \hat{b} + \hat{a}\hat{b}^\dagger\right)$$

$$\dot{O} = -\frac{i}{\hbar}\left[O, \tilde{H}\right] - \gamma O + \sqrt{2\gamma}O_{\text{in}}$$

$$\begin{aligned}\dot{\hat{a}} &= -i\omega_m\hat{a} - ig\hat{b} - \gamma_o\hat{a} + \sqrt{2\gamma_o}\hat{a}_{\text{in}}\\ \dot{\hat{b}} &= -i\omega_m\hat{b} - ig\hat{a} - \gamma_m\hat{b} + \sqrt{2\gamma_m}\hat{b}_{\text{in}}\end{aligned}$$

- Taking the Fourier transform [operators now implicitly function of frequency rather than time]

$$\begin{aligned}-i\omega\hat{a} &= -i\omega_m\hat{a} - ig\hat{b} - \gamma_o\hat{a} + \sqrt{2\gamma_o}\hat{a}_{\text{in}}\\ -i\omega\hat{b} &= -i\omega_m\hat{b} - ig\hat{a} - \gamma_m\hat{b} + \sqrt{2\gamma_m}\hat{b}_{\text{in}}\end{aligned}$$

Resolved sideband cooling

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- Rearrange for operators

$$\hat{a} = \frac{\sqrt{2\gamma_o}\hat{a}_{\text{in}} - ig\hat{b}}{\gamma_o + i(\omega_m - \omega)} = \frac{\sqrt{2\gamma_o}\hat{a}_{\text{in}} - ig\hat{b}}{\gamma_o + i\delta}$$

$$\delta = \omega_m - \omega$$

$$\hat{b} = \frac{\sqrt{2\gamma_m}\hat{b}_{\text{in}} - ig\hat{a}}{\gamma_m + i(\omega_m - \omega)} = \frac{\sqrt{2\gamma_m}\hat{b}_{\text{in}} - ig\hat{a}}{\gamma_m + i\delta}$$

- Solve simultaneously for *b*

$$\hat{b} = \sqrt{2\gamma_m}\left[\frac{\gamma_o + i\delta}{(\gamma_m + i\delta)(\gamma_o + i\delta) + g^2}\right]\hat{b}_{\text{in}} - \sqrt{2\gamma_o}\left[\frac{ig}{(\gamma_m + i\delta)(\gamma_o + i\delta) + g^2}\right]\hat{a}_{\text{in}}$$

decay rate and spectrum of *b* changed by interaction

optical driving of *b*

Short exercise: find *b* for yourself

Resolved sideband cooling

$$\hat{b} = \sqrt{2\gamma_m}\left[\frac{\gamma_o + i\delta}{(\gamma_m + i\delta)(\gamma_o + i\delta) + g^2}\right]\hat{b}_{\text{in}} - \sqrt{2\gamma_o}\left[\frac{ig}{(\gamma_m + i\delta)(\gamma_o + i\delta) + g^2}\right]\hat{a}_{\text{in}}$$

- $\hat{b}^\dagger(\omega)\hat{b}(\omega)$ proportional to mechanical oscillator energy at frequency ω.
- Hence mean phonon occupancy: $n \propto \int_{-\infty}^{\infty} \langle \hat{b}^\dagger \hat{b} \rangle d\omega$
- Assuming that the incident optical field is coherent, with *a* effectively being a vacuum state, $\langle \hat{a}_{\text{in}}^\dagger \hat{a}_{\text{in}} \rangle = 0$

$$\Rightarrow \langle \hat{b}^\dagger \hat{b} \rangle = \frac{2\gamma_m}{(\gamma_m^2 + \delta^2) + g^2\left(g^2 + 2\gamma_m\gamma_o - 2\delta^2\right)/\left(\gamma_o^2 + \delta^2\right)} \langle \hat{b}_{\text{in}}^\dagger \hat{b}_{\text{in}} \rangle$$

Short exercise: find this expression for yourself

Resolved sideband cooling

$$\hat{b} = \sqrt{2\gamma_m}\left[\frac{\gamma_o + i\delta}{(\gamma_m + i\delta)(\gamma_o + i\delta) + g^2}\right]\hat{b}_{\text{in}} - \sqrt{2\gamma_o}\left[\frac{ig}{(\gamma_m + i\delta)(\gamma_o + i\delta) + g^2}\right]\hat{a}_{\text{in}}$$

- $\hat{b}^\dagger(\omega)\hat{b}(\omega)$ proportional to mechanical oscillator energy at frequency ω.
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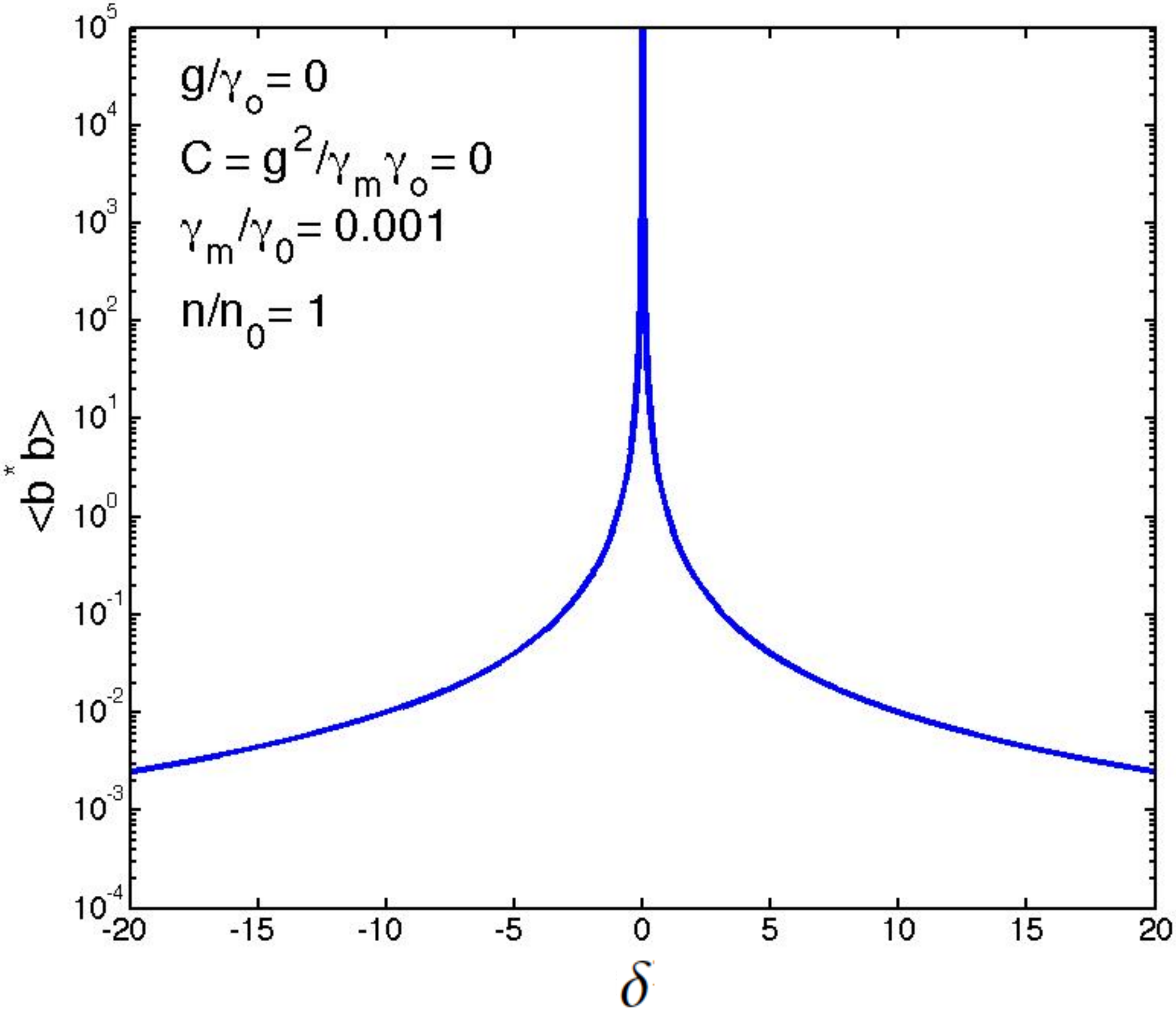
- Assuming that the incident optical field is coherent, with *a* effectively being a vacuum state, $\langle \hat{a}_{\text{in}}^\dagger \hat{a}_{\text{in}} \rangle = 0$

Uncoupled mechanical power spectrum

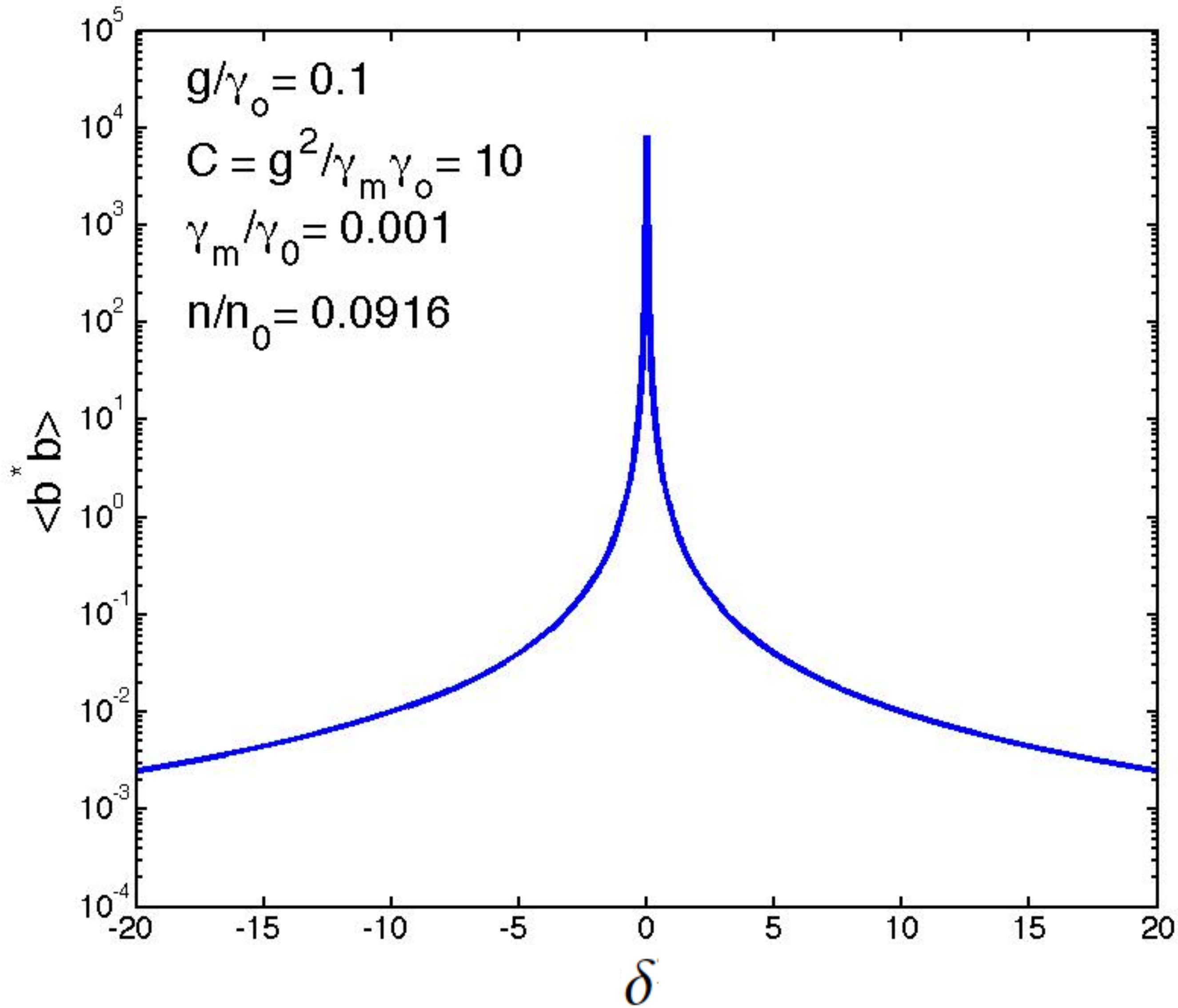
$$\langle \hat{b}^\dagger \hat{b} \rangle = \frac{\boxed{2\gamma_m}}{\boxed{(\gamma_m^2 + \delta^2)} + \boxed{g^2\left(g^2 + 2\gamma_m\gamma_o - 2\delta^2\right) / \left(\gamma_o^2 + \delta^2\right)}} \langle \hat{b}_{\text{in}}^\dagger \hat{b}_{\text{in}} \rangle$$

Optomechanical modification

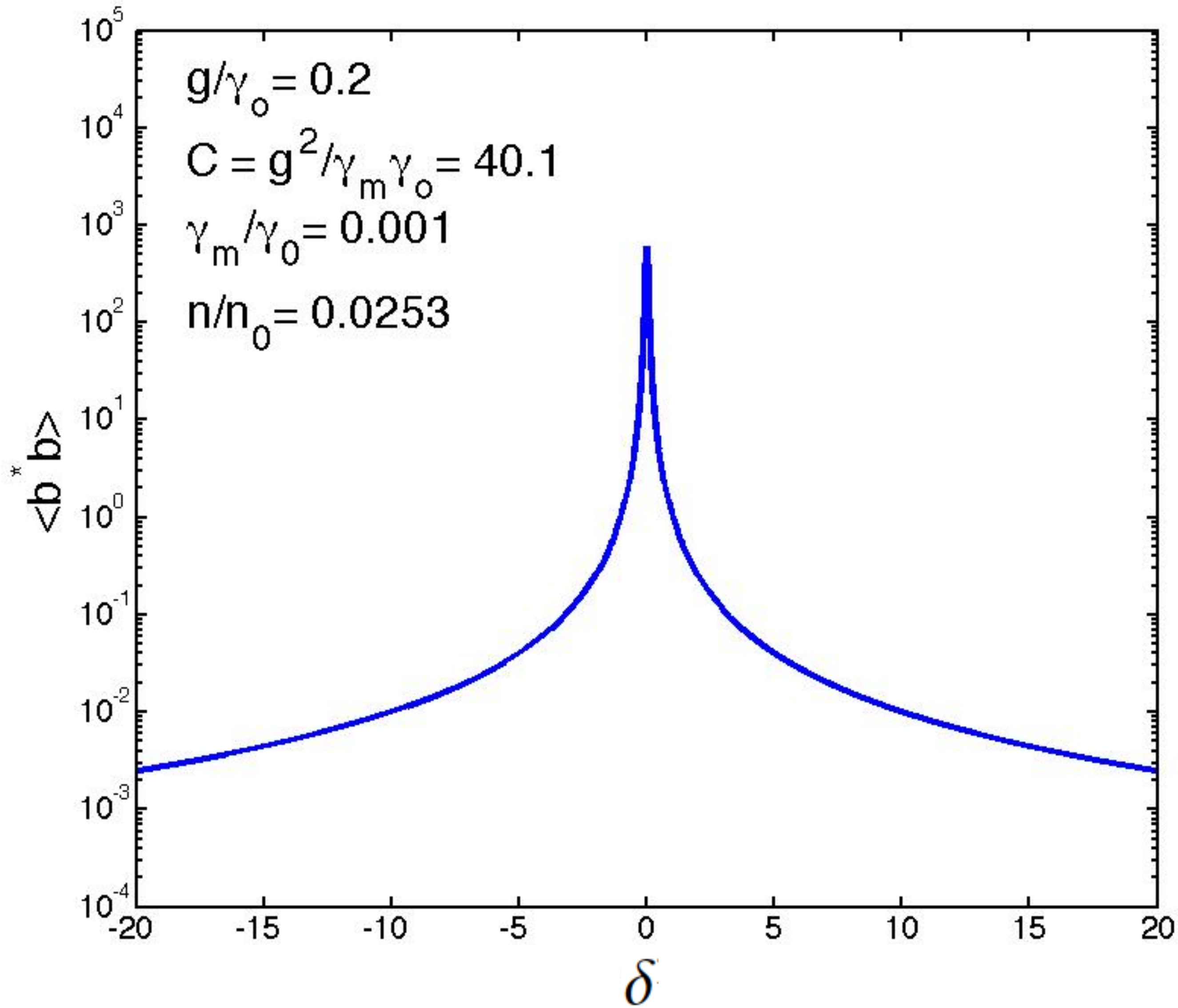
Resolved sideband cooling



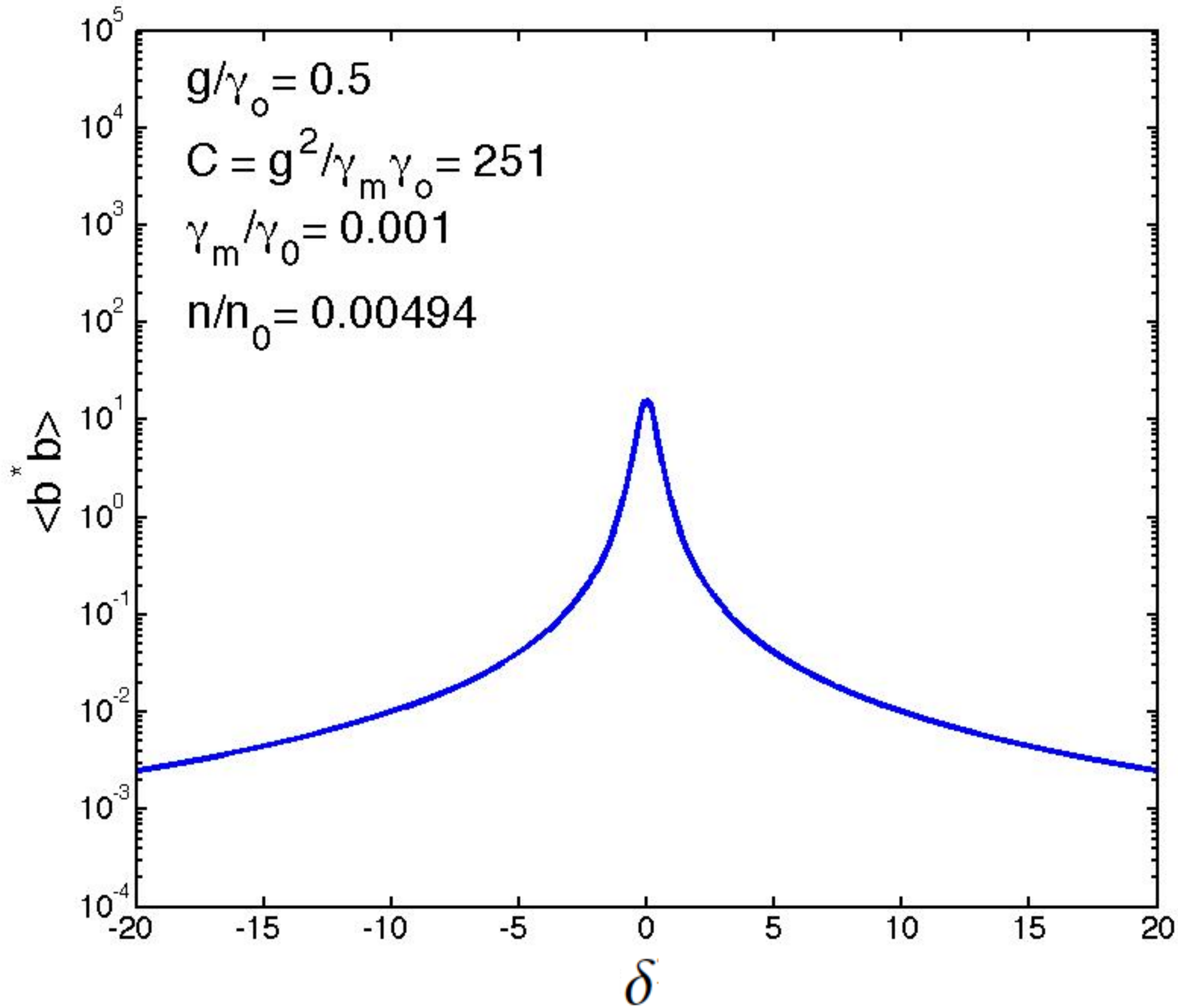
Resolved sideband cooling



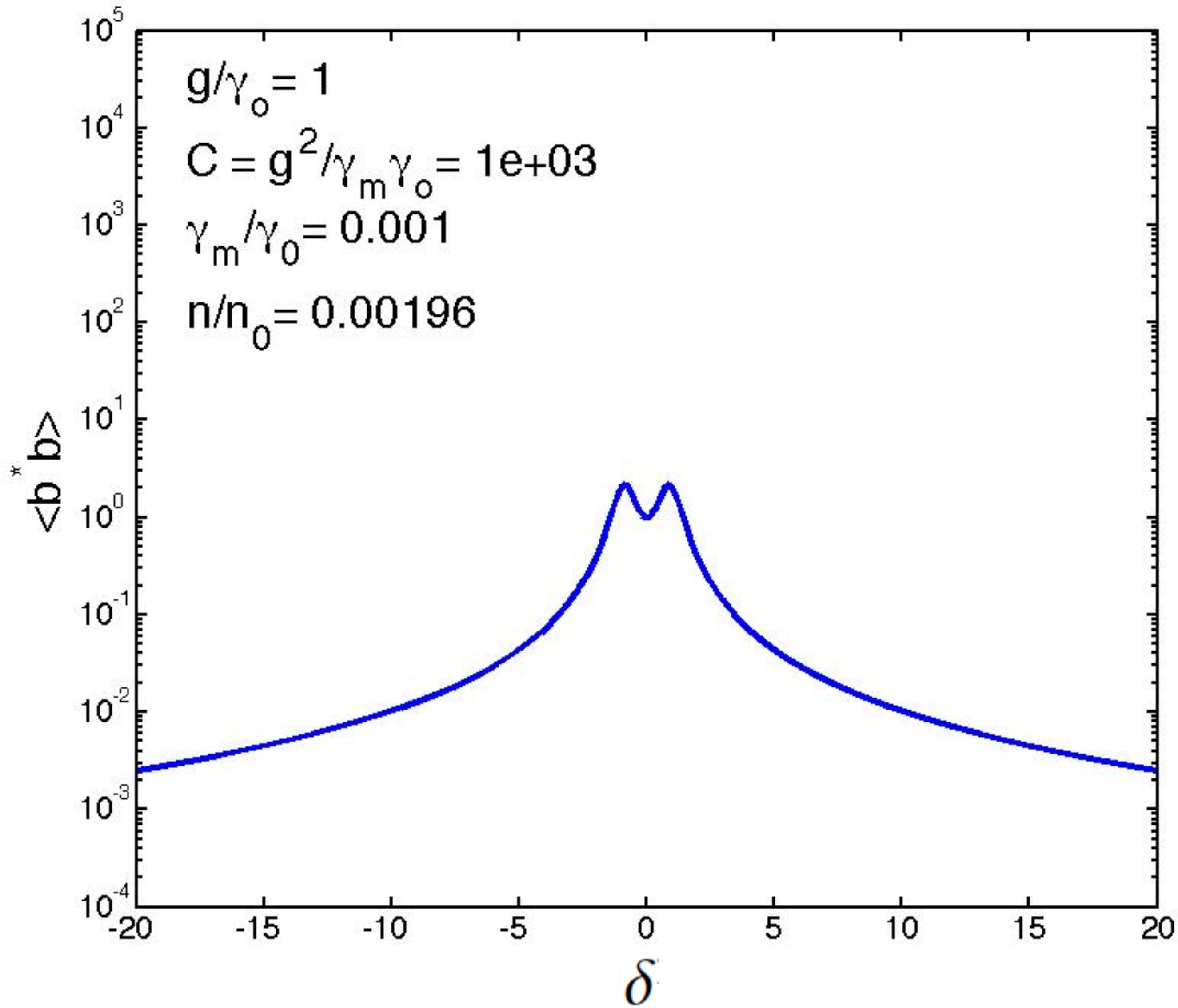
Resolved sideband cooling



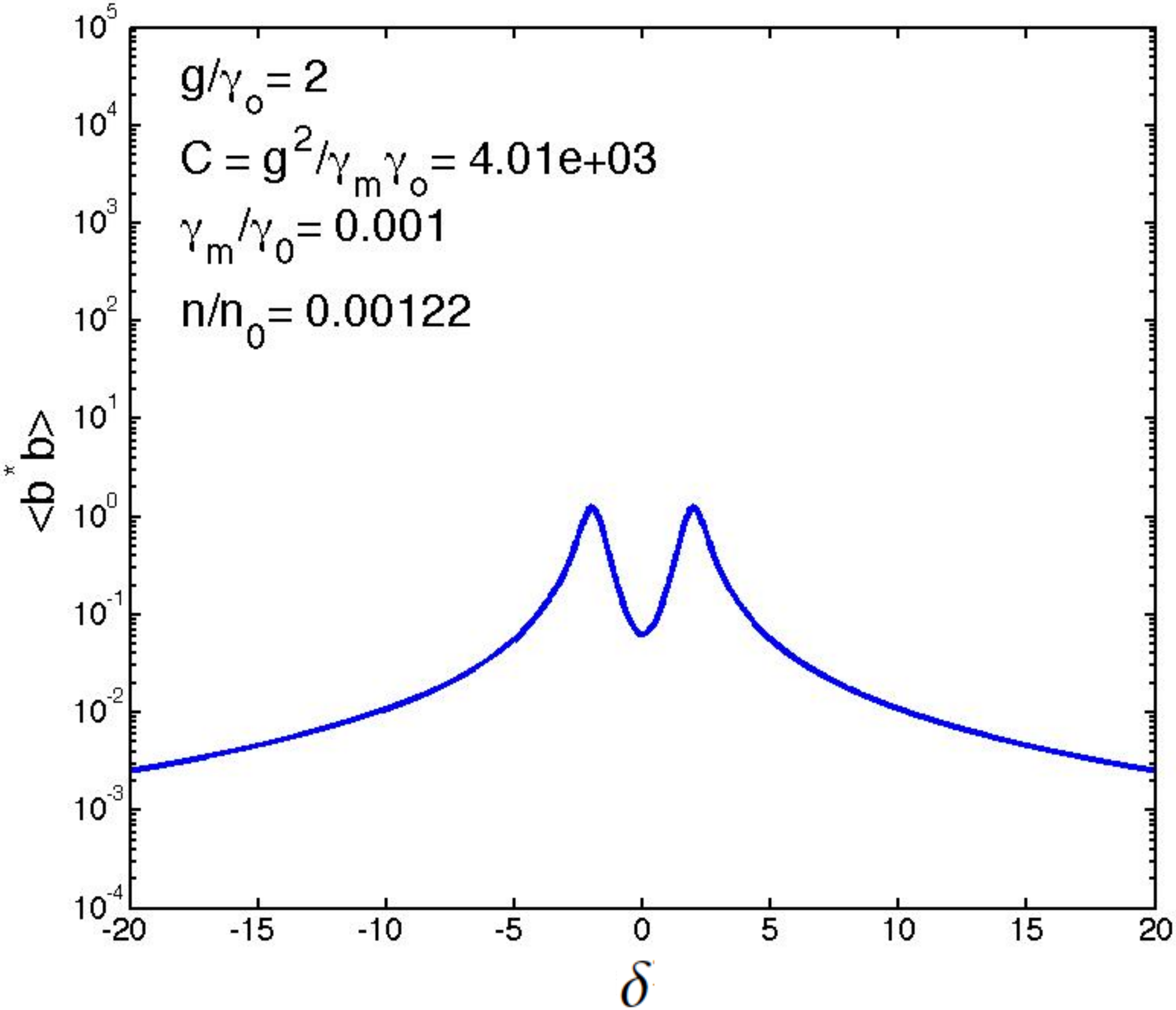
Resolved sideband cooling



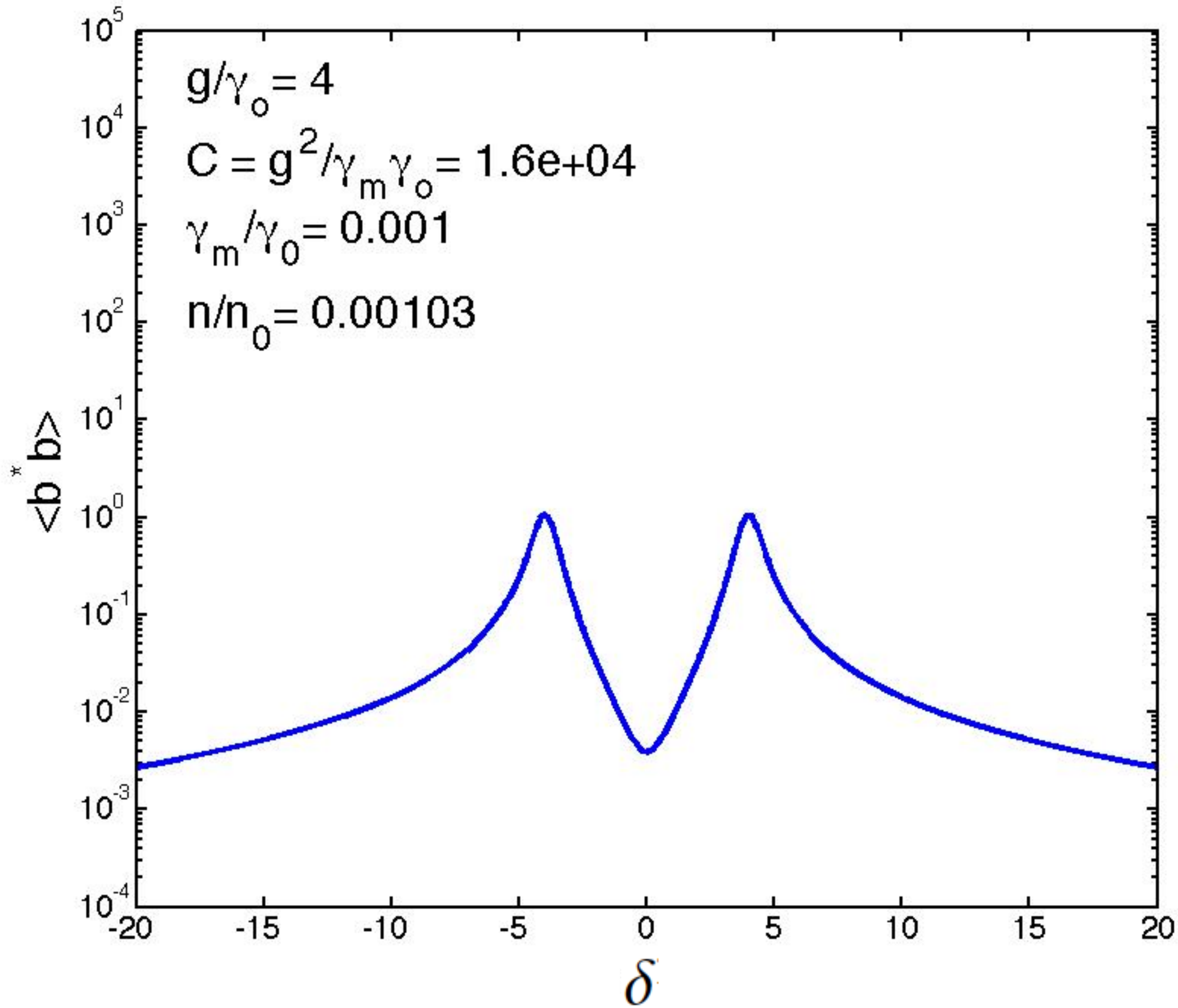
Resolved sideband cooling



Resolved sideband cooling



Resolved sideband cooling



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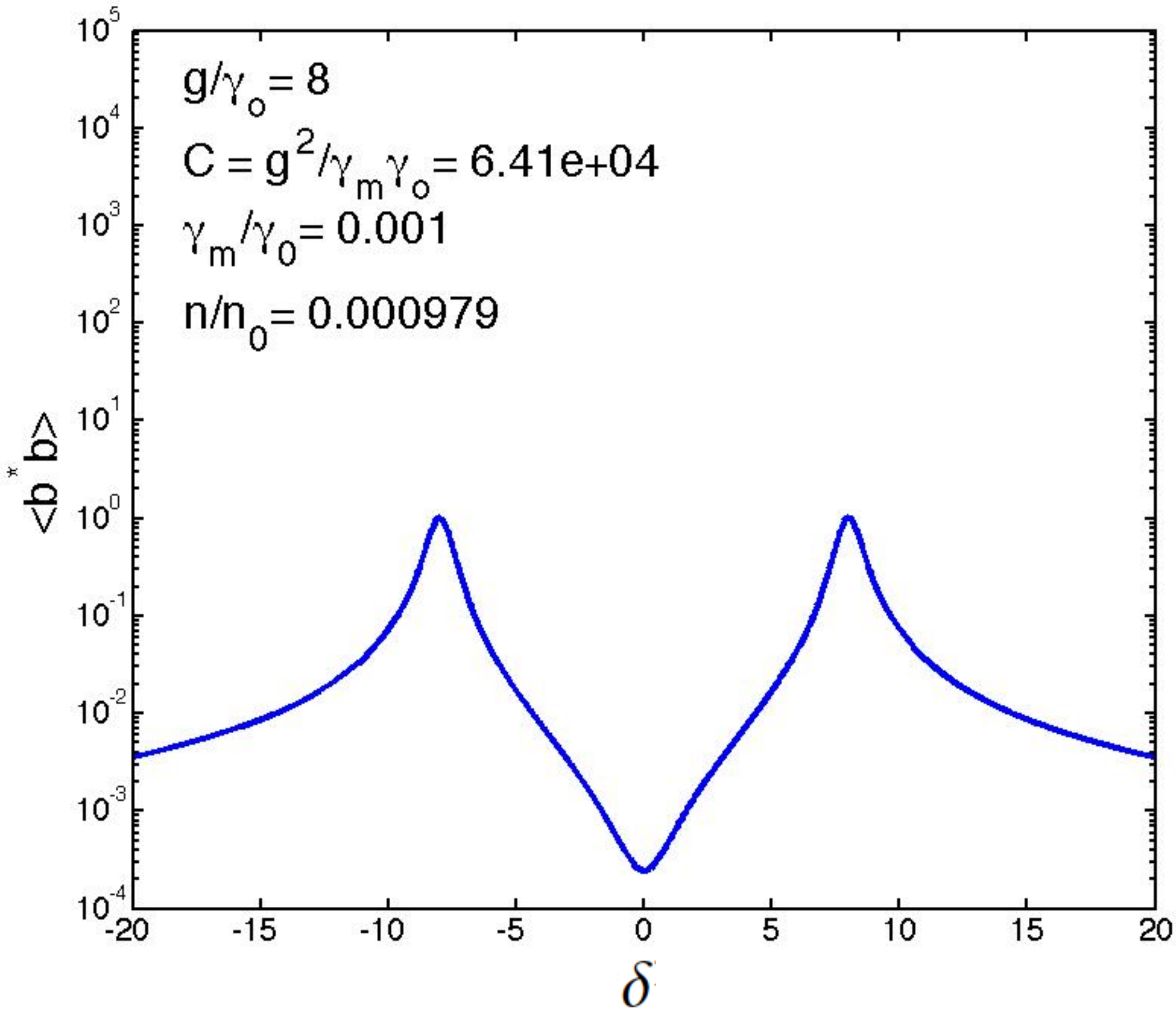
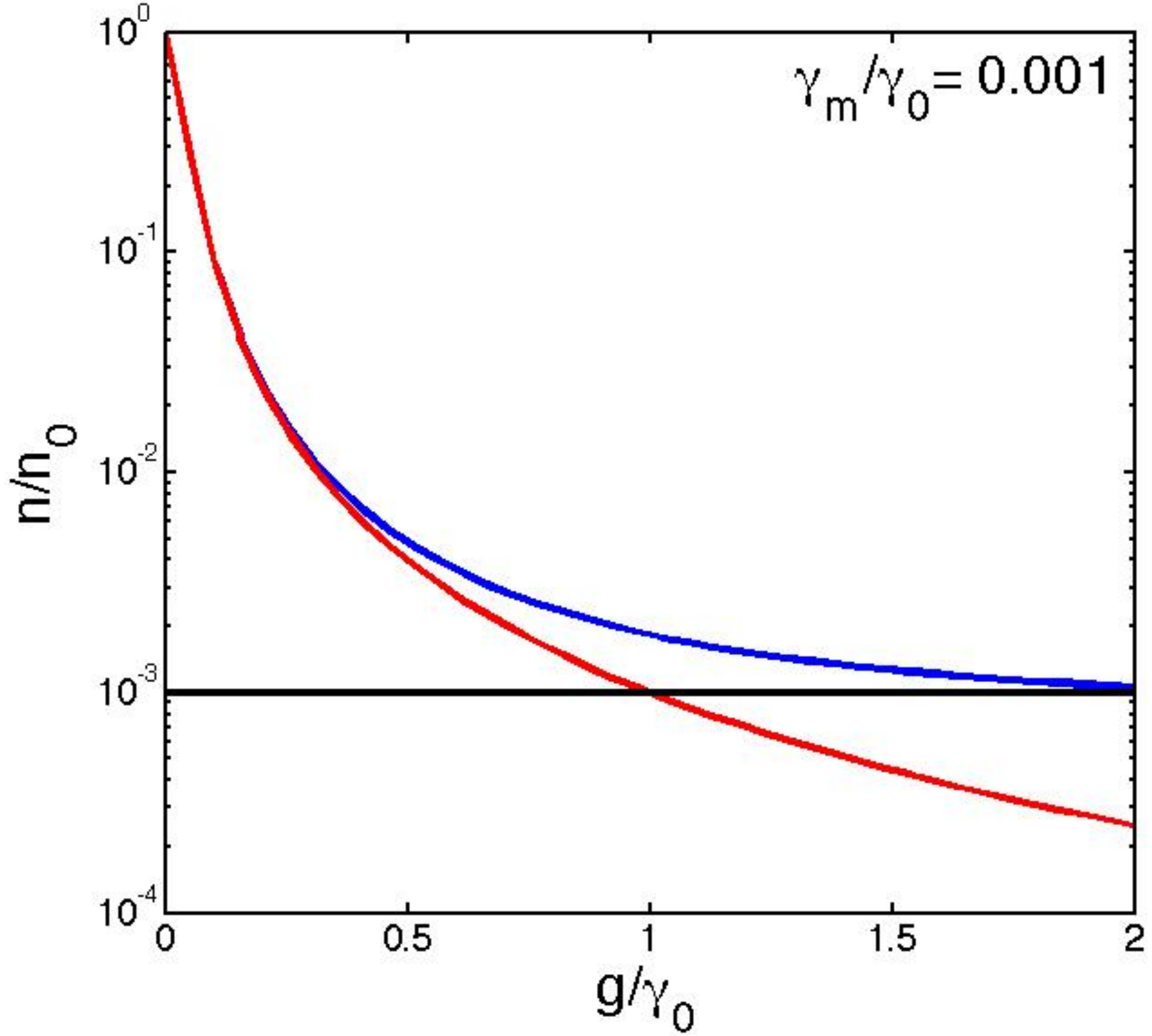


Resolved sideband cooling



Resolved sideband cooling

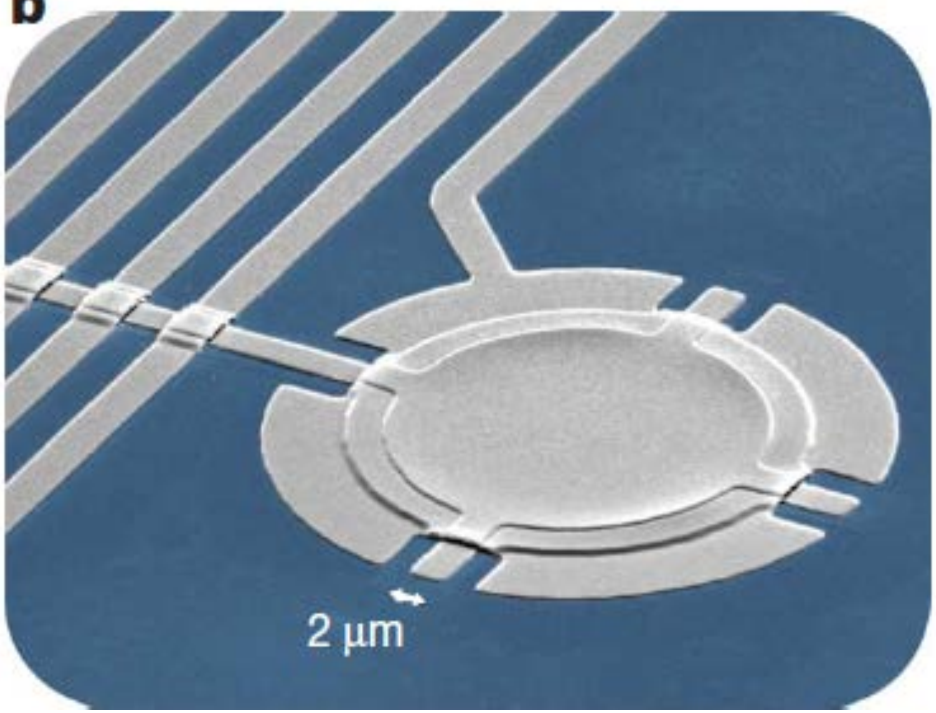
LETTER

doi:10.1038/nature09898

Circuit cavity electromechanics in the strong-coupling regime

J. D. Teufel[1], Dale Li[1], M. S. Allman[1], K. Cicak[1], A. J. Sirois[1], J. D. Whittaker[1] & R. W. Simmonds[1]

b



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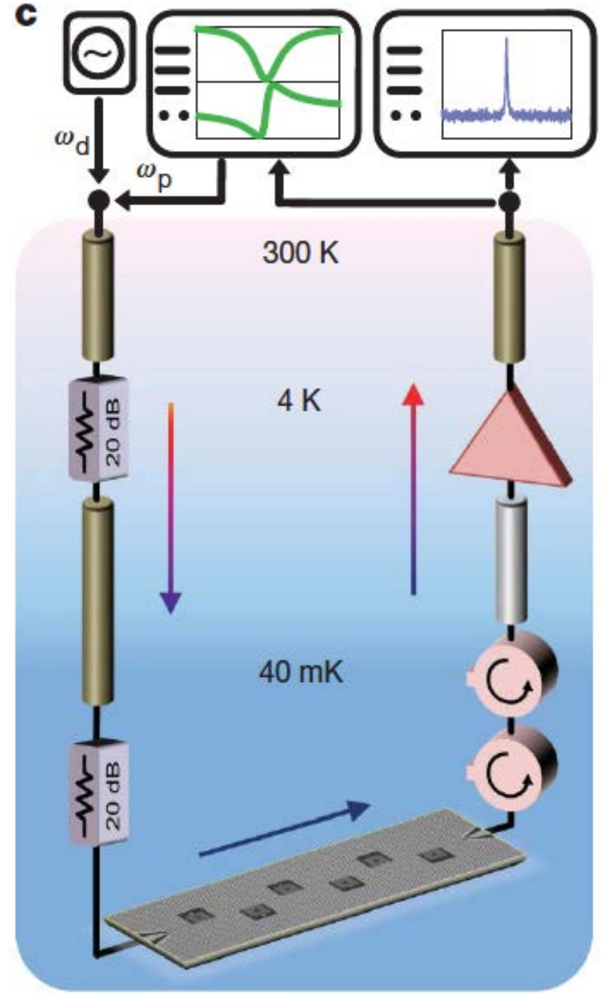

Resolved sideband cooling

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LETTER

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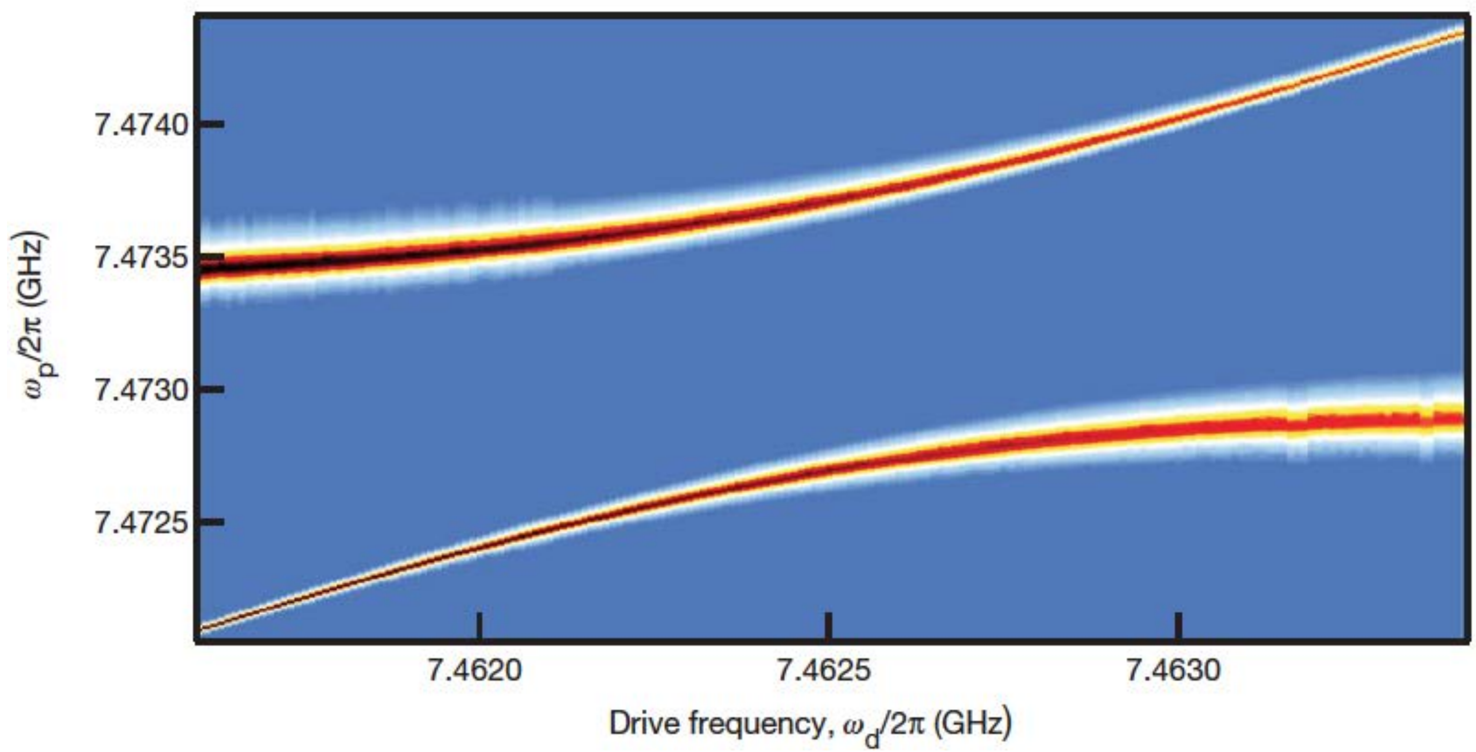


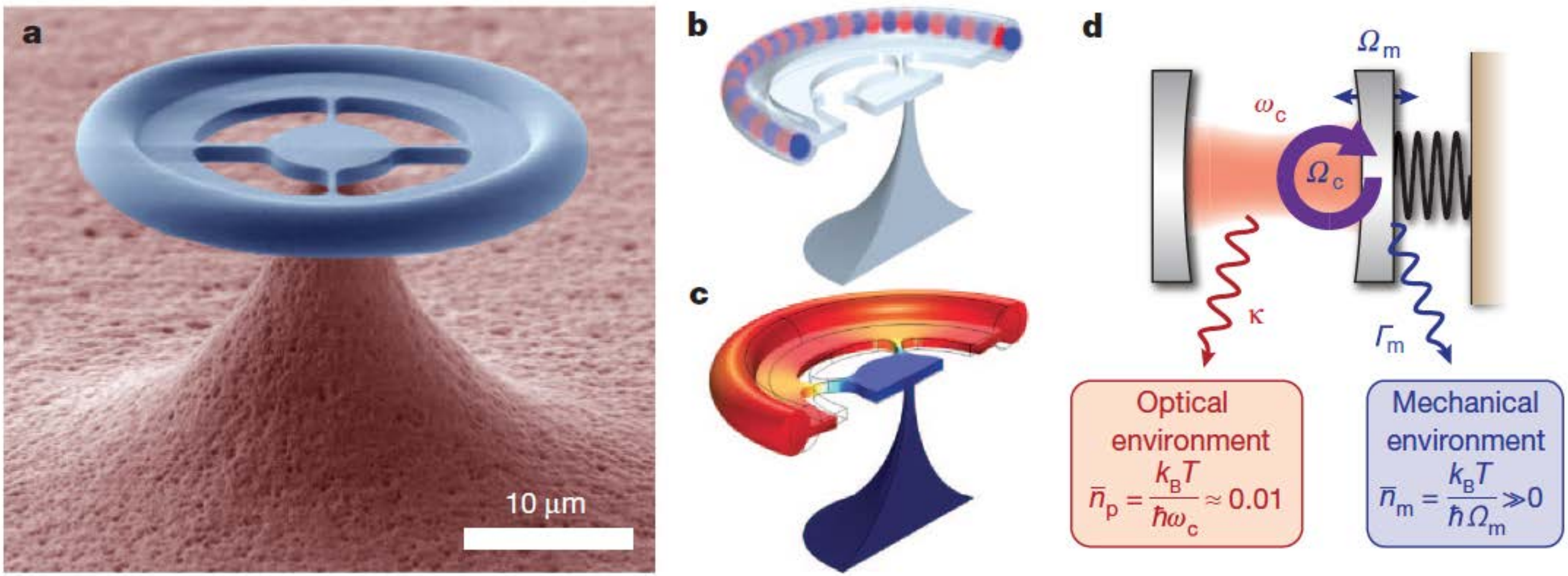
Resolved sideband cooling

LETTER

doi:10.1038/nature10787

Quantum-coherent coupling of a mechanical oscillator to an optical cavity mode

E. Verhagen[1]*, S. Deléglise[1]*, S. Weis[1,2]*, A. Schliesser[1,2]* & T. J. Kippenberg[1,2]



Resolved sideband cooling

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LETTER

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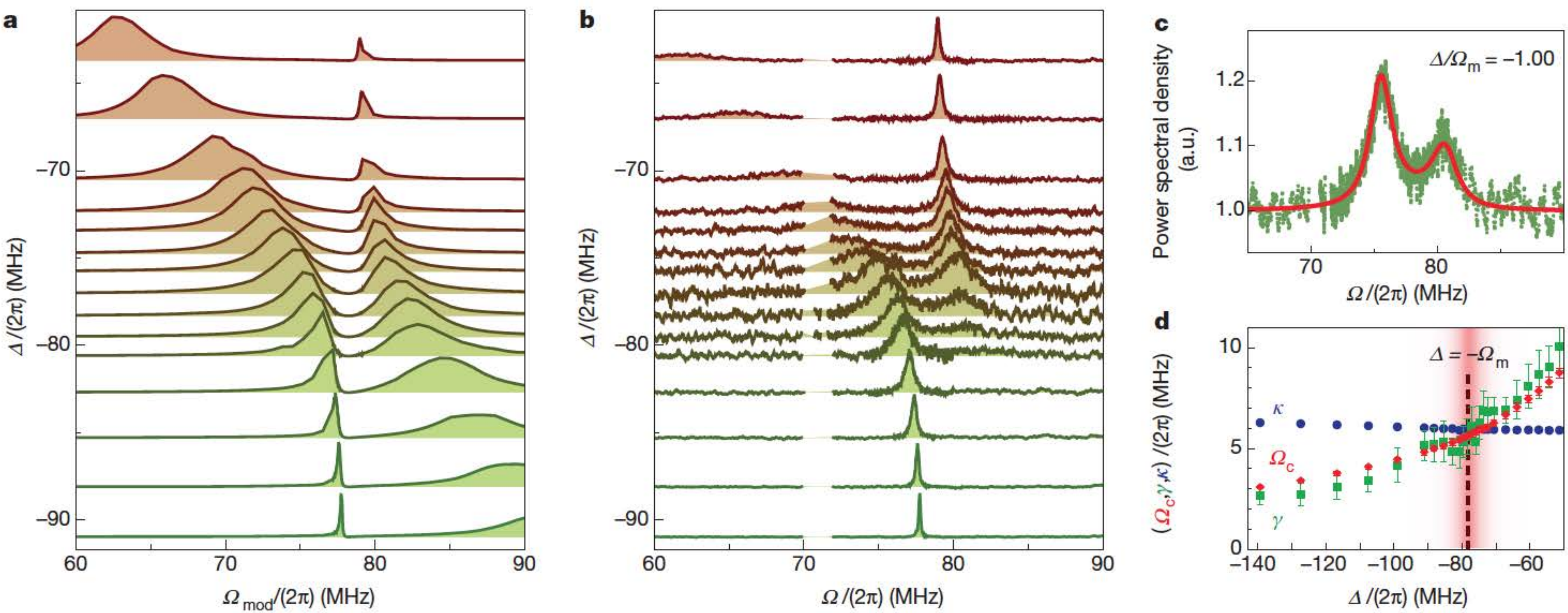

Resolved sideband cooling

$$\langle \hat{b}^\dagger \hat{b} \rangle = \frac{2\gamma_m}{(\gamma_m^2 + \delta^2) + g^2 (g^2 + 2\gamma_m\gamma_o - 2\delta^2) / (\gamma_o^2 + \delta^2)} \langle \hat{b}_{\mathrm{in}}^\dagger \hat{b}_{\mathrm{in}} \rangle$$

- In principle could (possibly) analytically integrate this expression to find the phonon occupancy.
- Much more tractable to consider two limits:
  - Weak optomechanical coupling regime with $\{g,\delta\} << \gamma_o$, where mechanical spectrum appears to be a modified Lorenzian.
  - Strong optomechanical coupling regime with $g >> \gamma_o$ where mechanical spectrum appears to be a double-Lorenzian.

Possibly impossible exercise: find an analytical general expression for n

Resolved sideband cooling

- Trick: use approximations to re-express $\langle \hat{b}^\dagger \hat{b} \rangle$ in the form of a (or a pair of) Lorenzian(s)
- Weak coupling regime:

$$\langle \hat{b}^\dagger \hat{b} \rangle = \frac{2\gamma_m}{\gamma_m^2 (1 + C)^2 + \delta^2} \langle \hat{b}_{\rm in}^\dagger \hat{b}_{\rm in} \rangle$$

$$C = \frac{g^2}{\gamma_m \gamma_o}$$

Optomechanical cooperativity

- Strong coupling regime:

$$\langle \hat{b}^\dagger \hat{b} \rangle = \frac{\gamma_m/2}{\left(\frac{\gamma_m+\gamma_o}{2}\right)^2 + \delta^2} \langle \hat{b}_{\rm in}^\dagger \hat{b}_{\rm in} \rangle$$

[Here $\delta \rightarrow g + \delta$ and δ << *g*]

Reasonable exercise: derive each of these expressions

Resolved sideband cooling

- Trick: use approximations to re-express $\langle \hat{b}^\dagger \hat{b} \rangle$ in the form of a (or a pair of) Lorenzian(s)
- Weak coupling regime:

$$\langle \hat{b}^\dagger \hat{b} \rangle = \frac{2\gamma_m}{\gamma_m^2 (1 + C)^2 + \delta^2} \langle \hat{b}^\dagger_{\rm in} \hat{b}_{\rm in} \rangle \qquad C = \frac{g^2}{\gamma_m \gamma_o}$$

Modified peak of Lorenzian

- Strong coupling regime:

$$\langle \hat{b}^\dagger \hat{b} \rangle = \frac{\gamma_m/2}{\left(\frac{\gamma_m + \gamma_o}{2}\right)^2 + \delta^2} \langle \hat{b}^\dagger_{\rm in} \hat{b}_{\rm in} \rangle$$

[Here $\delta \longrightarrow g + \delta$ and $\delta << g$]

Reasonable exercise: derive each of these expressions

Resolved sideband cooling

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- Trick: use approximations to re-express $\langle \hat{b}^\dagger \hat{b} \rangle$ in the form of a (or a pair of) Lorenzian(s)
- Weak coupling regime:

$$\langle \hat{b}^\dagger \hat{b} \rangle = \frac{2\gamma_m}{\boxed{\gamma_m^2 (1+C)^2} + \delta^2} \langle \hat{b}_{\text{in}}^\dagger \hat{b}_{\text{in}} \rangle \qquad C = \frac{g^2}{\gamma_m \gamma_o}$$

Square of modified linewidth (dissipation rate)

- Strong coupling regime:

$$\langle \hat{b}^\dagger \hat{b} \rangle = \frac{\gamma_m/2}{\boxed{\left(\frac{\gamma_m+\gamma_o}{2}\right)^2} + \delta^2} \langle \hat{b}_{\text{in}}^\dagger \hat{b}_{\text{in}} \rangle$$

[Here $\delta \to g + \delta$ and $\delta \ll g$]

Reasonable exercise: derive each of these expressions

Resolved sideband cooling

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- Rather than integrating each Lorenzian to find the phonon occupancy, we recognise that the area under a Lorenzian is proportional to its width times it's height.
- Consequently

Modified width

Modified peak

$$\frac{n}{n_{g=0}} = \left(\frac{\boxed{\gamma}}{\boxed{\gamma_{g=0}}}\right) \times \frac{\boxed{\langle \hat{b}^\dagger(\omega_m)\hat{b}(\omega_m)\rangle}}{\boxed{\langle \hat{b}^\dagger(\omega_m)\hat{b}(\omega_m)\rangle_{g=0}}}$$

Bare width

Bare peak

Resolved sideband cooling

- We then find optomechanical coupling reduced occupancies of….
- Weak coupling regime:

$$\frac{n}{n_{g=0}} = \frac{\gamma_m \gamma_o}{g^2 + \gamma_m \gamma_o} = \boxed{\frac{1}{C+1}} \qquad C = \frac{g^2}{\gamma_m \gamma_o}$$

Occupancy scales ~ as g^{-2}

- Strong coupling regime:

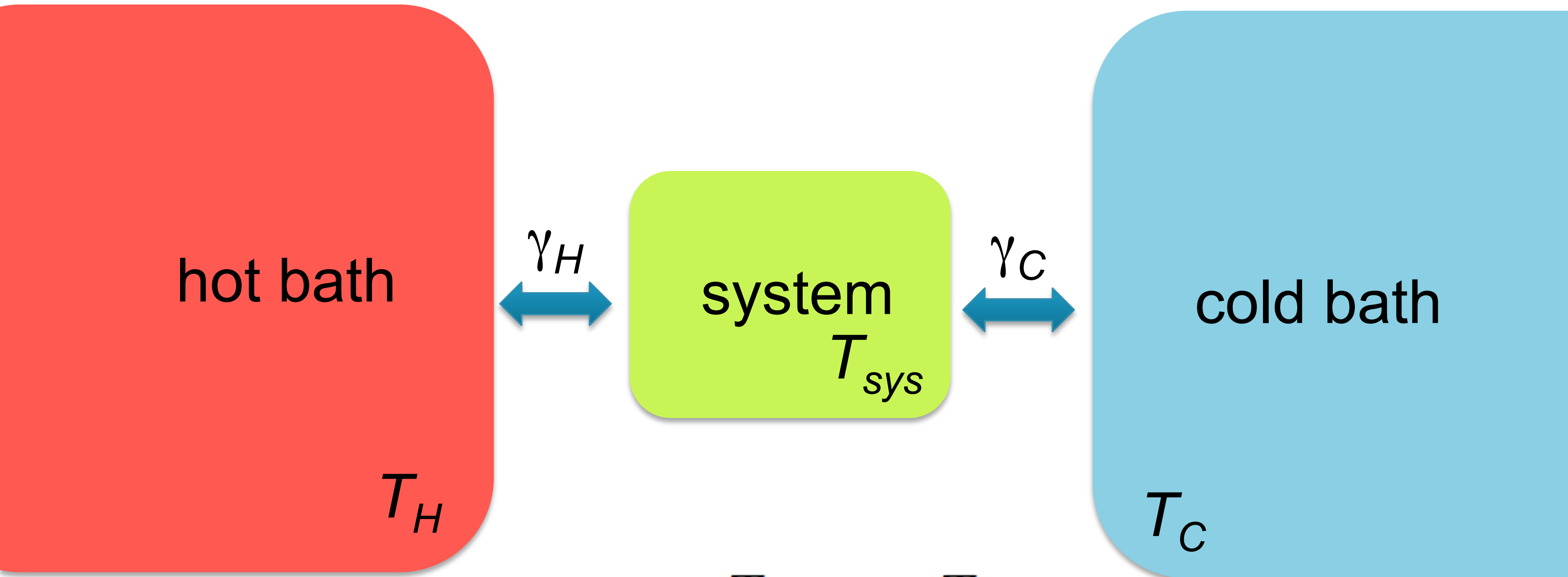
$$\frac{n_{g \gg \{\gamma_o, \gamma_m\}}}{n_{g=0}} = \left(\frac{\gamma_m}{\gamma_m + \gamma_o}\right) \approx \boxed{\frac{\gamma_m}{\gamma_o}}$$

Occupancy only depends on ratio of dissipation rates

Easy exercise: show that these results are correct

Thermodynamic understanding of cooling

- These cooling predictions can be reproduced from a simple thermodynamical model.



$$T_{\text{system}} = \frac{T_H \gamma_H + T_C \gamma_C}{\gamma_H + \gamma_C}$$

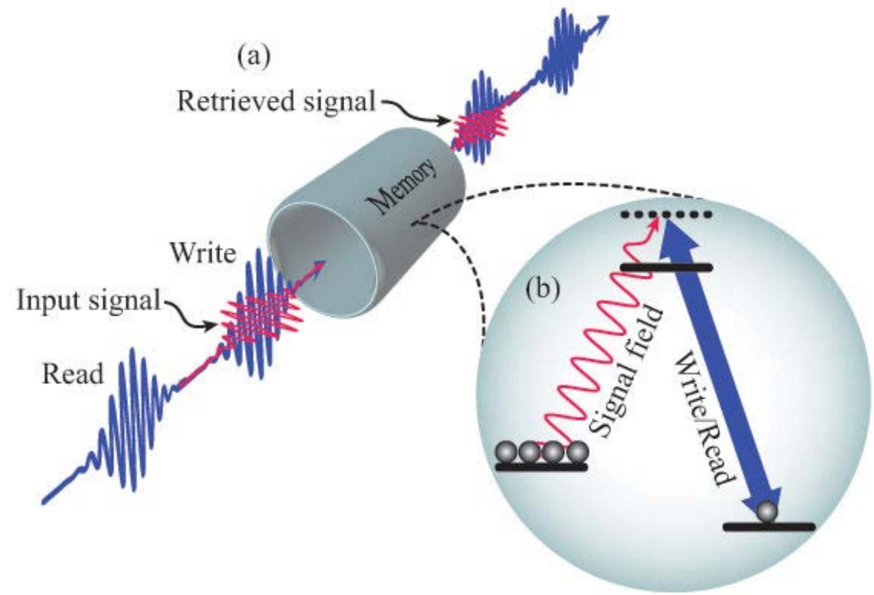
Thermodynamic understanding of cooling

- The coupling rate to the hot bath is simply the bare mechanical dissipation rate.
- In the strong coupling regime the coupling rate to the cold bath (the light) is just the optical dissipation rate.
  - It doesn’t matter how fast the coupling is between mechanics and intracavity field, the bottleneck is the rate heat leaves the cavity.
- In the weak coupling regime, on the other hand, the bottleneck is the coupling from mechanics to intracavity field, with rate given by C.
- Setting T_C=0, and substituting the rates above, retrieves identical final phonon occupancies.

Exercise: show this yourself, and justify why C is the correct rate to use in the weak coupling regime.

Optomechanically induced transparency

Electromagnetically induced transparency



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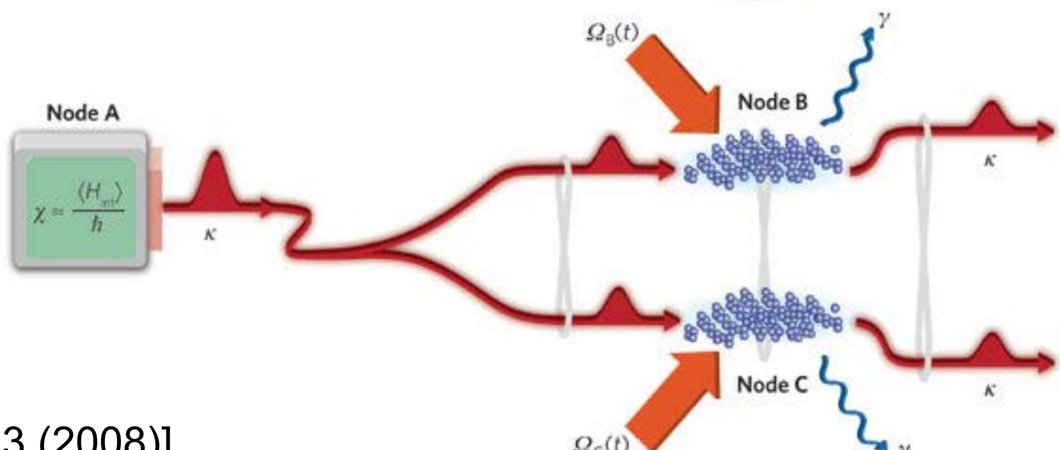
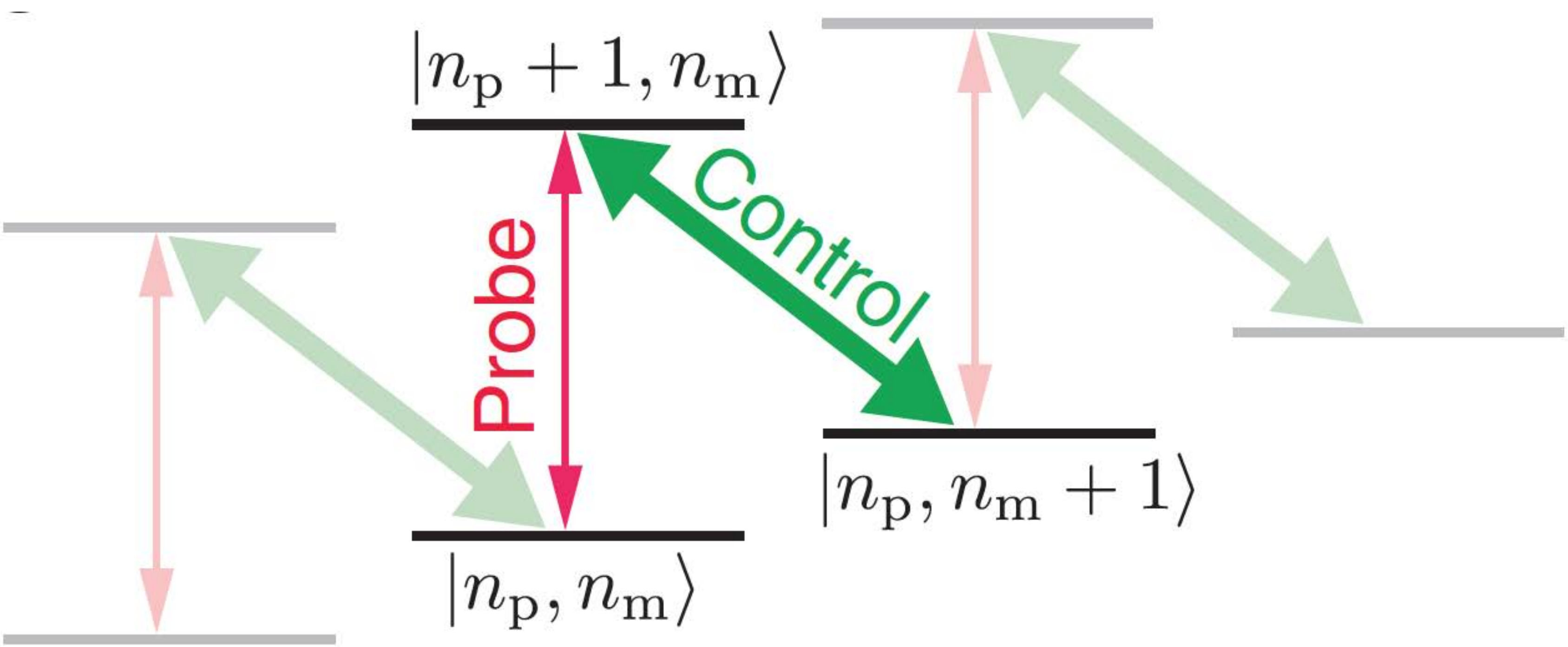


[Kimble, Nature **453** 1023 (2008)]

Optomechanically induced transparency

Optomechanically Induced Transparency
Stefan Weis *et al.*
Science **330**, 1520 (2010);
DOI: 10.1126/science.1195596



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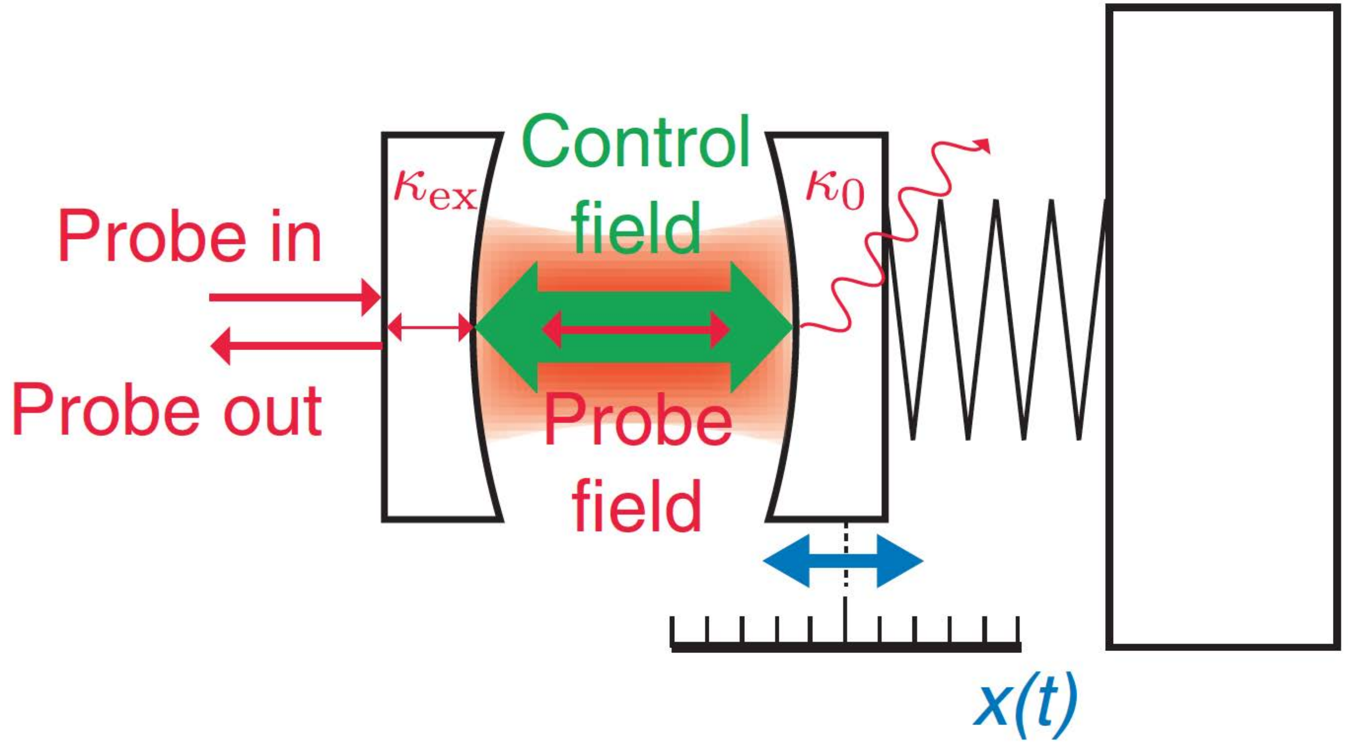

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Optomechanically Induced Transparency
Stefan Weis *et al.*
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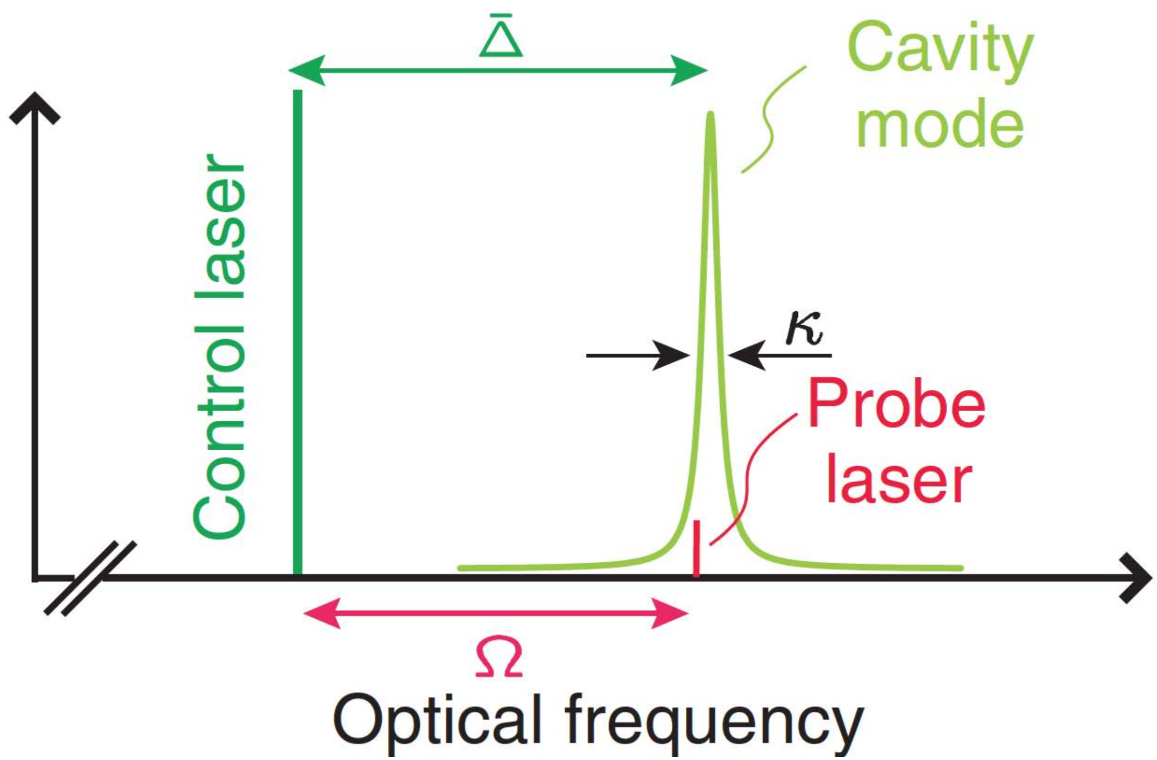

Optomechanically induced transparency

- Generic optomechanical Hamiltonian:

$$\tilde{H} = \hbar\boxed{\Delta}\hat{a}^\dagger\hat{a} + \hbar\omega_m\hat{b}^\dagger\hat{b} + \hbar g_0\hat{a}^\dagger\hat{a}\left(\hat{b}^\dagger + \hat{b}\right)$$

Set Δ=0 ➔ rotating frame at cavity resonance frequency

Optomechanically induced transparency

- Generic optomechanical Hamiltonian:

$$\tilde{H} = \hbar\omega_m \hat{b}^\dagger \hat{b} + \hbar g_0 \boxed{\hat{a}^\dagger \hat{a}} \left(\hat{b}^\dagger + \hat{b}\right)$$

Includes both strong classical *control field* and *probe field*

Optomechanically induced transparency

- Generic optomechanical Hamiltonian:

$$\tilde{H} = \hbar\omega_m \hat{b}^\dagger \hat{b} + \hbar g_0 \hat{a}^\dagger \hat{a}\left(\hat{b}^\dagger + \hat{b}\right)$$

$$\hat{a} \rightarrow \boxed{\hat{a}} + \boxed{\alpha e^{i\omega_m t}}$$

Probe field Classical control field

DC term: no effect on dynamics

Quantum noise driving: only relevant in strong single photon driving limit

$$\hat{a}^\dagger \hat{a} \;\rightarrow\; \cancel{\alpha^2} + \cancel{\hat{a}^\dagger \hat{a}} + \alpha\left[\hat{a}^\dagger e^{i\omega_m t} + \hat{a} e^{-i\omega_m t}\right]$$

$$\approx \;\alpha\left[\hat{a}^\dagger e^{i\omega_m t} + \hat{a} e^{-i\omega_m t}\right]$$

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$$\tilde{H} = \hbar\omega_m \hat{b}^\dagger \hat{b} + \hbar \boxed{g} \left[\hat{a}^\dagger e^{i\omega_m t} + \hat{a} e^{-i\omega_m t}\right] \left(\hat{b}^\dagger + \hat{b}\right)$$

Coherent amplitude boosted coupling rate $g = g_0\alpha$

Optomechanically induced transparency

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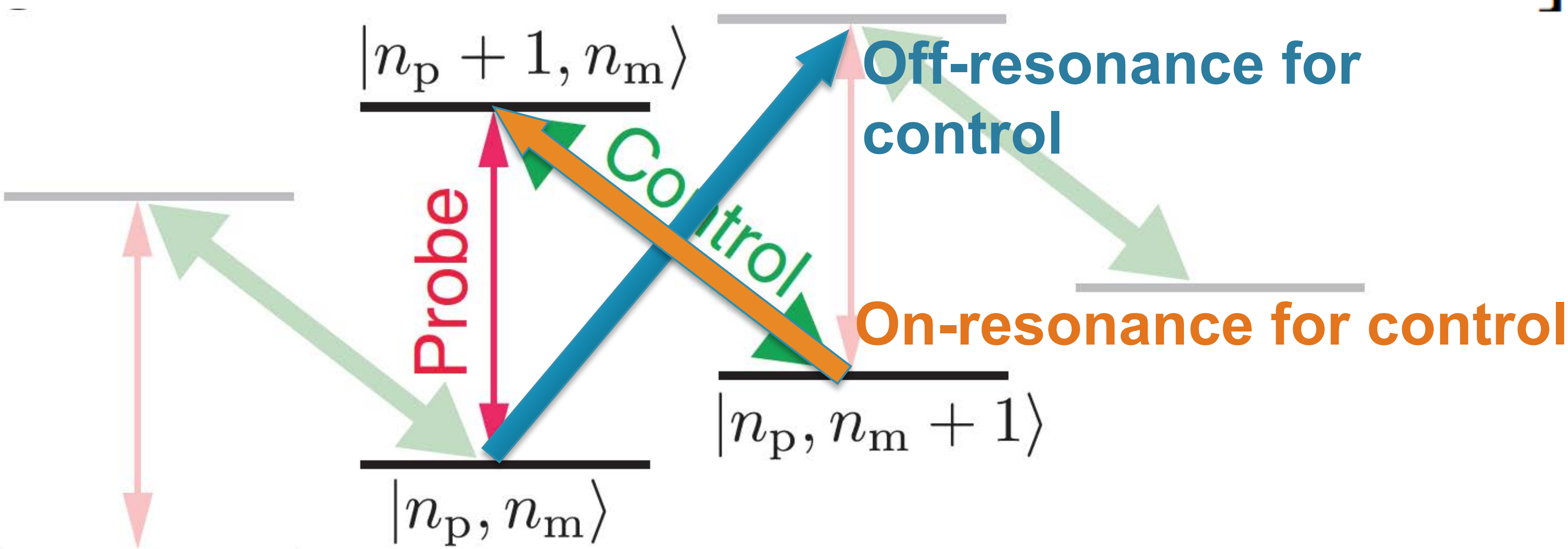
$$
\begin{aligned}
\tilde{H} &= \hbar\omega_m \hat{b}^\dagger \hat{b} + \hbar g \left[\hat{a}^\dagger e^{i\omega_m t} + \hat{a} e^{-i\omega_m t}\right]\left(\hat{b}^\dagger + \hat{b}\right) \\
&= \hbar\omega_m \hat{b}^\dagger \hat{b} + \hbar g \left[\cancel{\hat{a}^\dagger \hat{b}^\dagger e^{i\omega_m t}} + \boxed{\hat{a}\hat{b}^\dagger} e^{-i\omega_m t} \right. \\
&\qquad \left. + \boxed{\hat{a}^\dagger \hat{b}} e^{i\omega_m t} + \cancel{\hat{a}\hat{b} e^{-i\omega_m t}}\right]
\end{aligned}
$$



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$$\tilde{H} = \hbar\omega_m \hat{b}^\dagger \hat{b} + \hbar g \left[\hat{a}^\dagger e^{i\omega_m t} + \hat{a} e^{-i\omega_m t}\right]\left(\hat{b}^\dagger + \hat{b}\right)$$

$$= \hbar\omega_m \hat{b}^\dagger \hat{b} + \hbar g \left[\cancel{\hat{a}^\dagger \hat{b}^\dagger e^{i\omega_m t}} + \boxed{\hat{a}\hat{b}^\dagger} e^{-i\omega_m t} + \boxed{\hat{a}^\dagger \hat{b}} e^{i\omega_m t} + \cancel{\hat{a}\hat{b} e^{-i\omega_m t}}\right]$$

- OMIT Hamiltonian:

$$\boxed{\tilde{H} \approx \hbar\omega_m \hat{b}^\dagger \hat{b} + \hbar g \left[\hat{a}\hat{b}^\dagger e^{-i\omega_m t} + \hat{a}^\dagger \hat{b} e^{i\omega_m t}\right]}$$

Long exercise: determine the behaviour of OMIT including off-resonant terms

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$$\tilde{H} \approx \hbar\omega_m \hat{b}^\dagger \hat{b} + \hbar g\left[\hat{a}\hat{b}^\dagger e^{-i\omega_m t} + \hat{a}^\dagger \hat{b} e^{i\omega_m t}\right]$$

- Quantum Langevin equation…

$$\dot{\hat{a}} = -\gamma_o \hat{a} - ig\hat{b}e^{i\omega_m t} + \sqrt{2\gamma_o}\hat{a}_{\text{in}}$$
$$\dot{\hat{b}} = -\gamma_m \hat{b} - i\omega_m b - ig\hat{a}e^{-i\omega_m t} + \sqrt{2\gamma_m}\hat{b}_{\text{in}}$$

- Solve via Fourier transform (again):

$$-i\omega\hat{a}(\omega) = -\gamma_o \hat{a}(\omega) - ig\hat{b}(\omega+\omega_m) + \sqrt{2\gamma_0}\hat{a}_{\text{in}}(\omega)$$
$$-i\omega\hat{b}(\omega) = -(\gamma_m + i\omega_m)\,\hat{b}(\omega) - ig\hat{a}(\omega-\omega_m) + \sqrt{2\gamma_m}\hat{b}_{\text{in}}(\omega)$$

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$$-i\omega\hat{a}(\omega) = -\gamma_o\hat{a}(\omega) - ig\hat{b}(\omega+\omega_m) + \sqrt{2\gamma_0}\hat{a}_{\mathrm{in}}(\omega)$$

$$-i\omega\hat{b}(\omega) = -(\gamma_m + i\omega_m)\,\hat{b}(\omega) - ig\hat{a}(\omega-\omega_m) + \sqrt{2\gamma_m}\hat{b}_{\mathrm{in}}(\omega)$$

$$\hat{b}(\omega+\omega_m) = (\gamma_m - i\omega)^{-1}\left[-ig\hat{a}(\omega) + \sqrt{2\gamma_m}\hat{b}_{\mathrm{in}}(\omega+\omega_m)\right]$$

$$\hat{a}(\omega) = \underbrace{\chi(\omega)}_{\text{Optical susceptibility}}\left[\underbrace{-\sqrt{2\gamma_m}\left(\frac{ig}{\gamma_m - i\omega}\right)\hat{b}_{\mathrm{in}}(\omega+\omega_m)}_{\text{Fluctuations from mechanics}} + \underbrace{\sqrt{2\gamma_o}\hat{a}_{\mathrm{in}}(\omega)}_{\text{Fluctuations from light}}\right]$$

$$\chi(\omega)^{-1} = \gamma_o - i\omega + \frac{g^2}{\gamma_m - i\omega}$$

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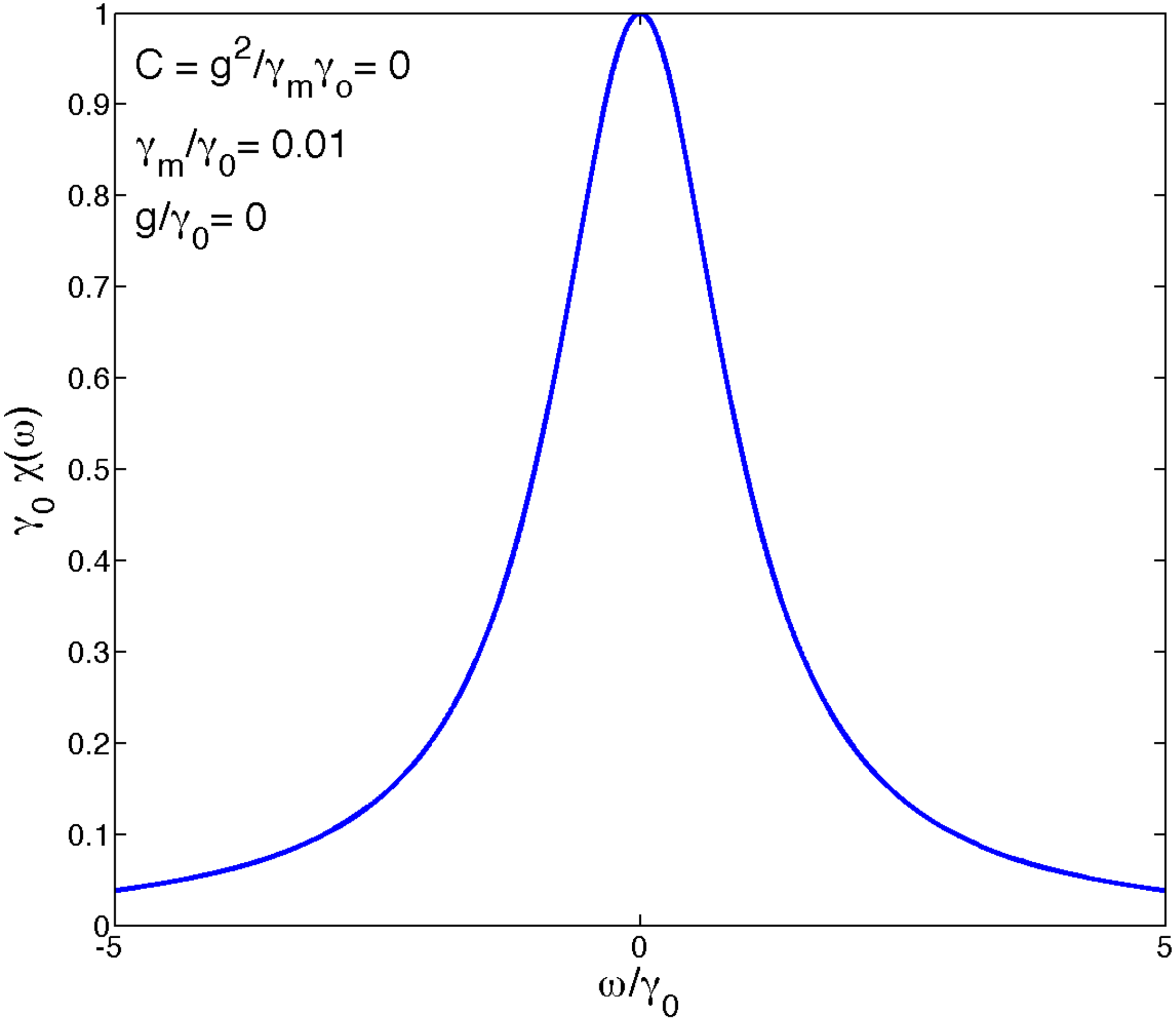
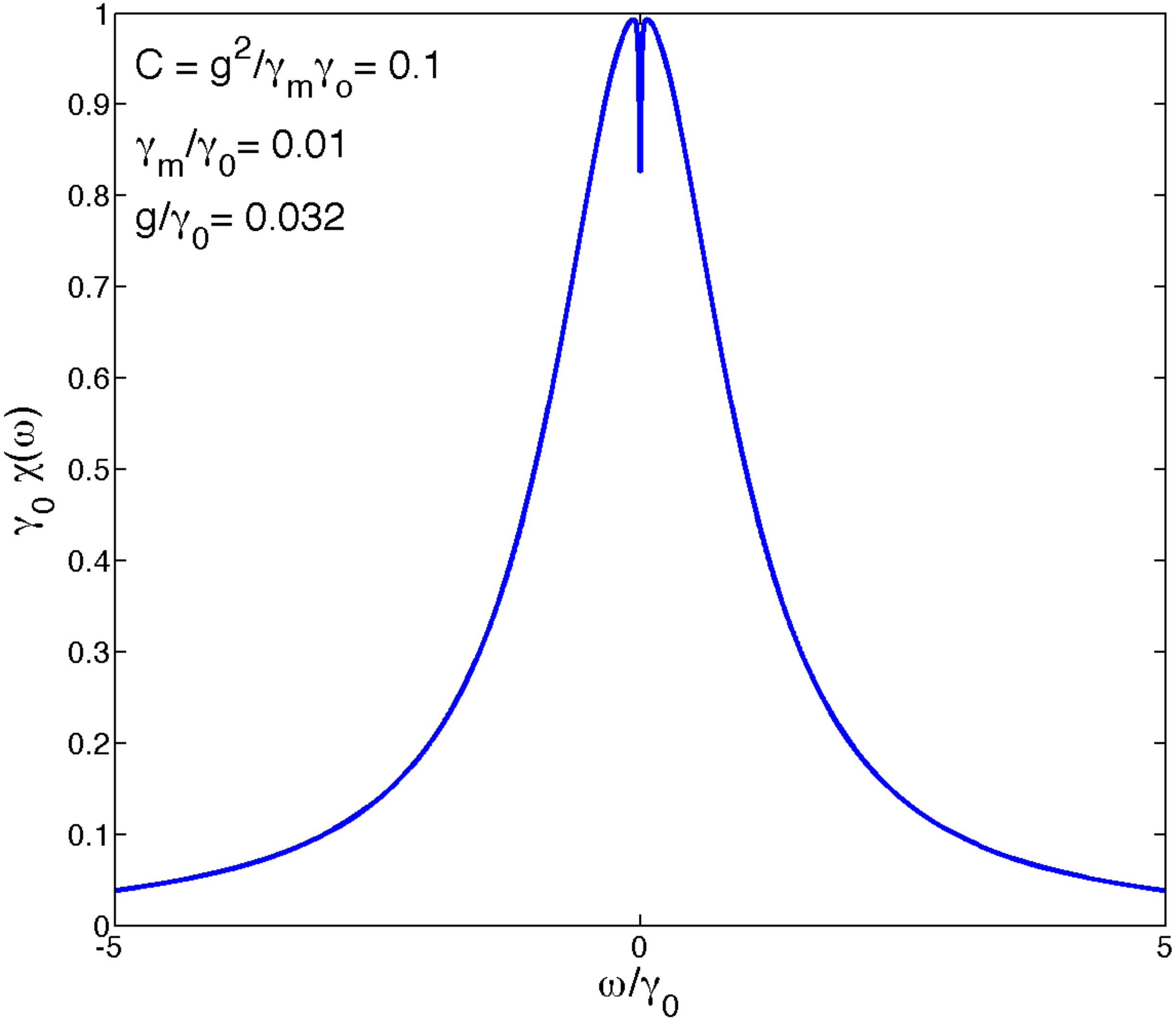
Optical susceptibility χ



Optomechanically induced transparency

Optical susceptibility χ



Optomechanically induced transparency

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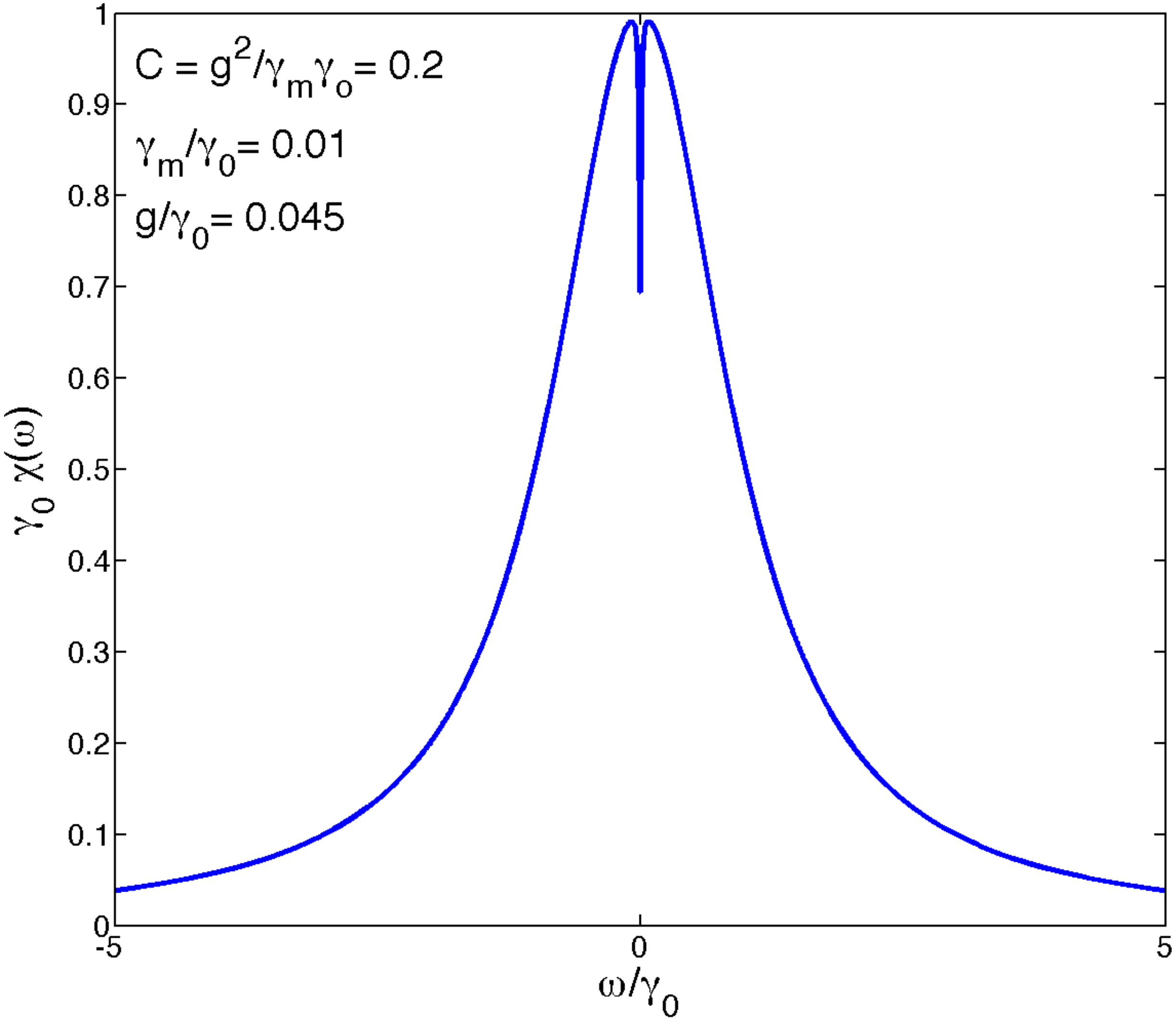

Optomechanically induced transparency

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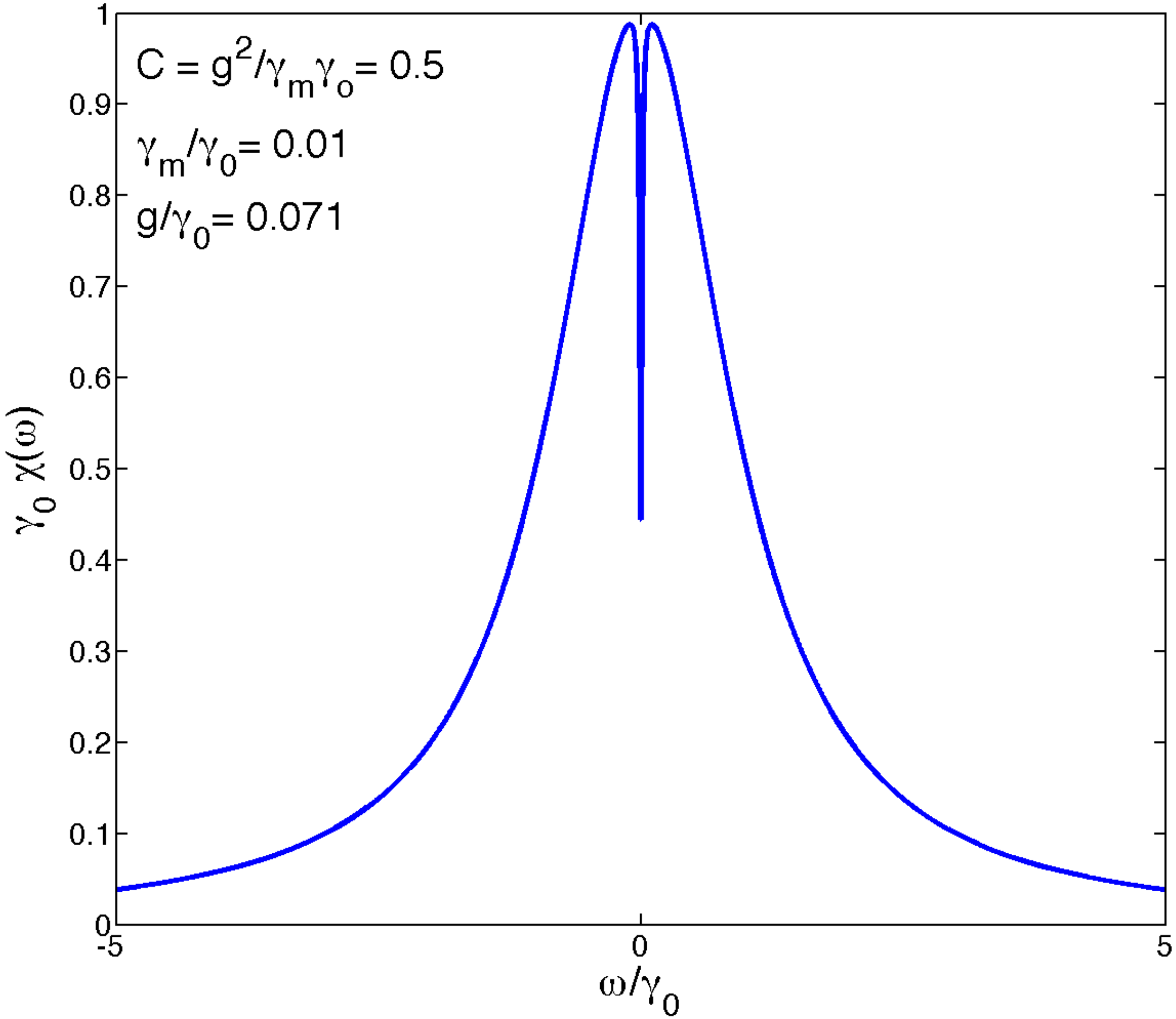

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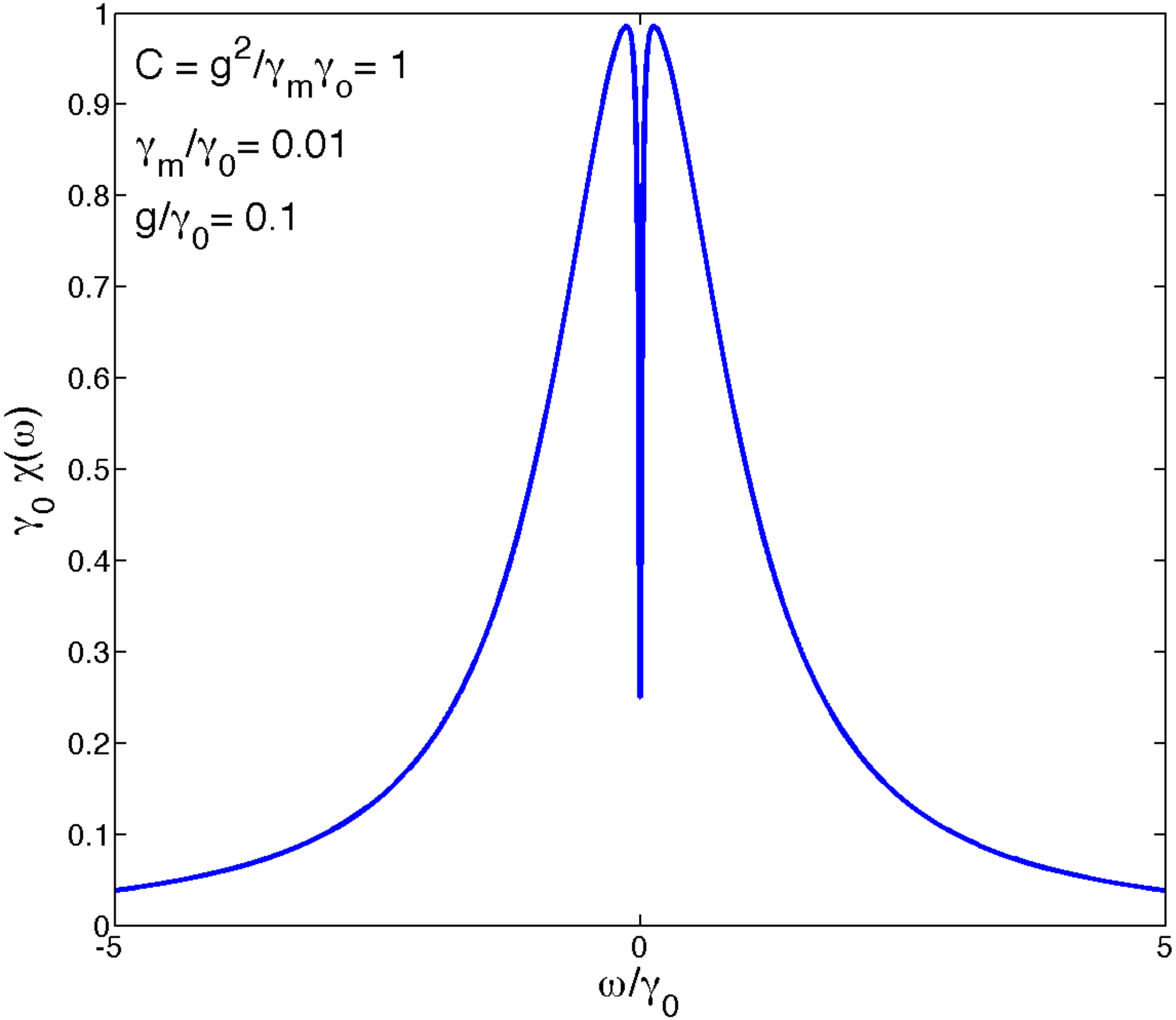

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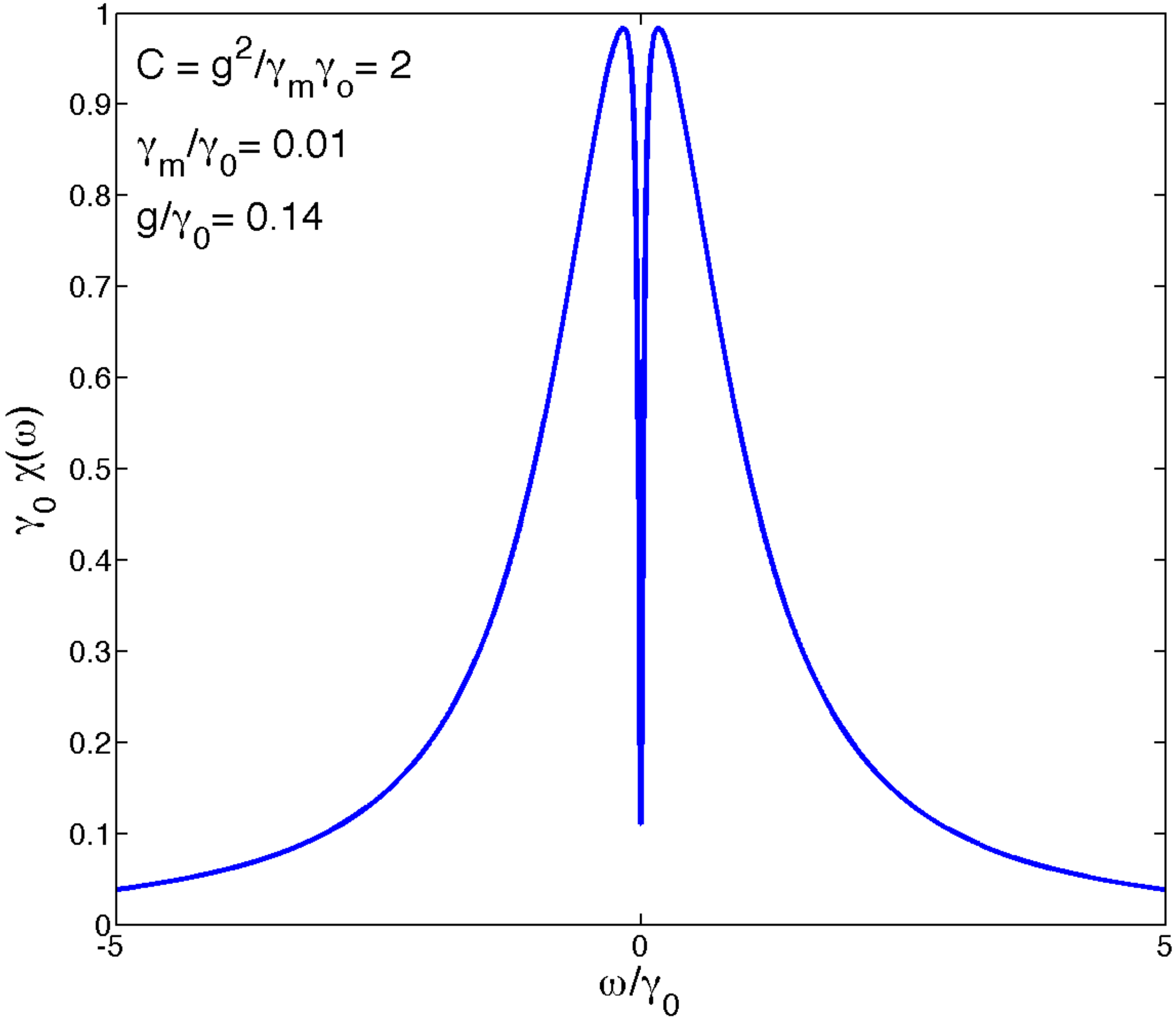

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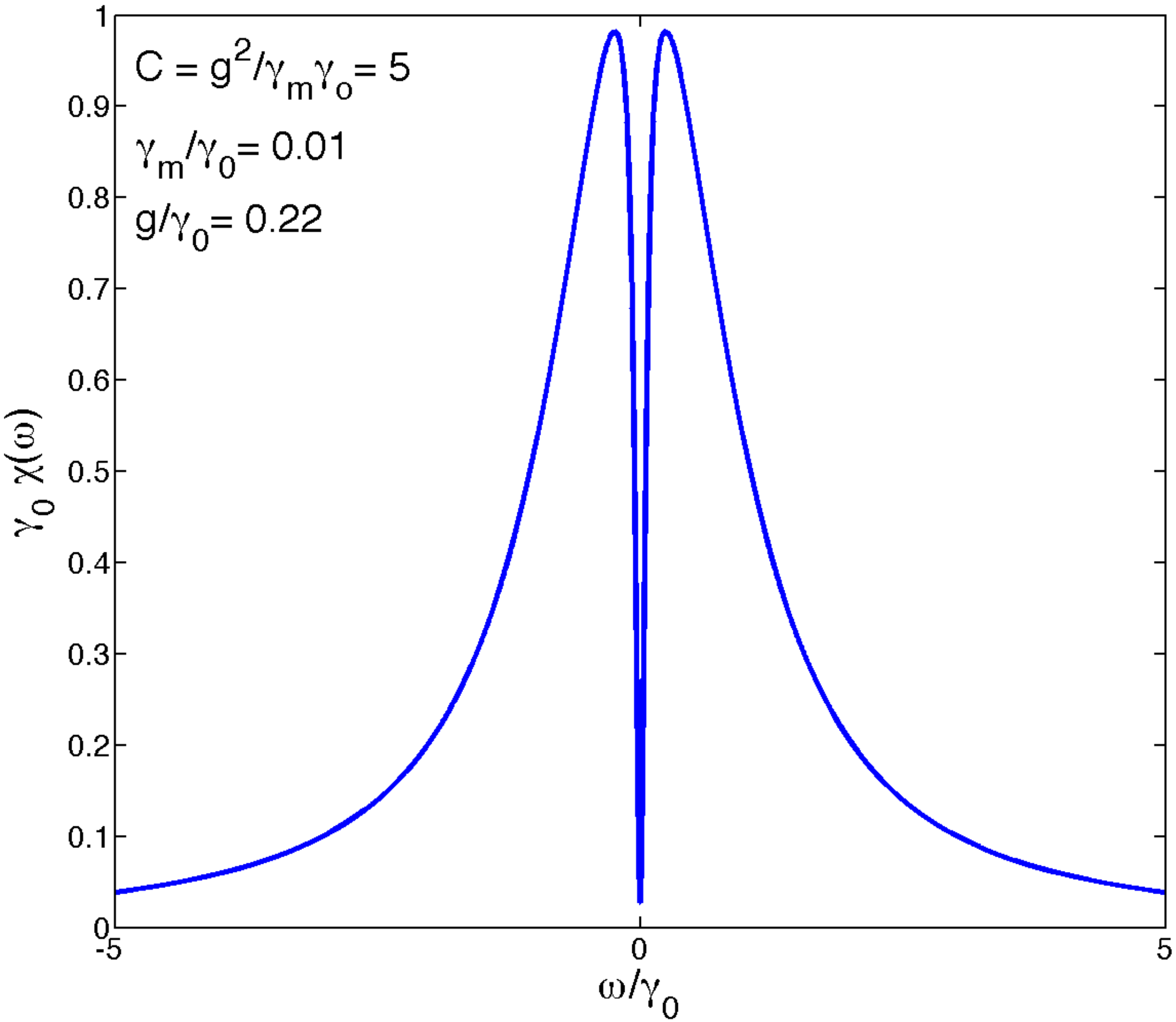

Optomechanically induced transparency

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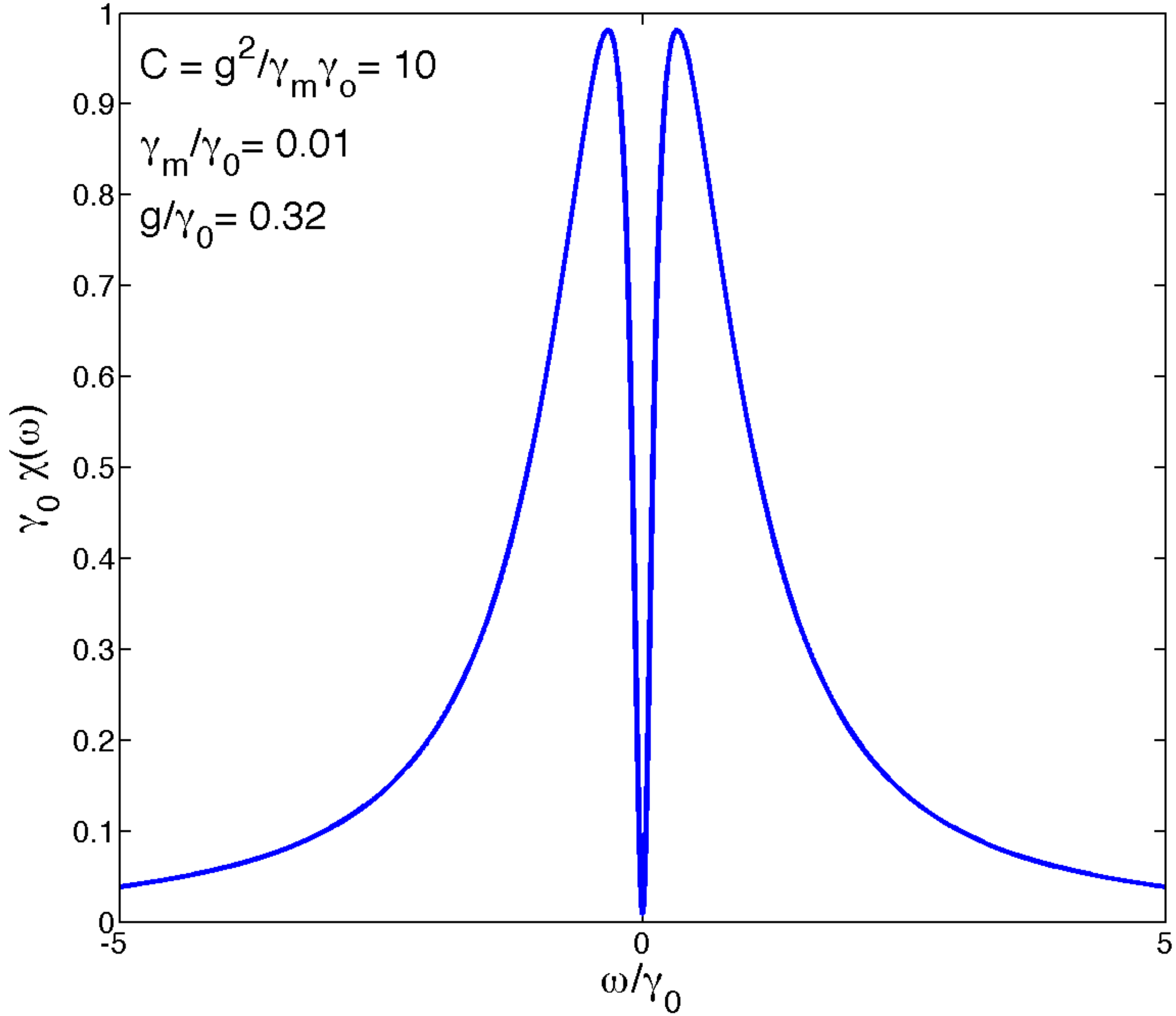
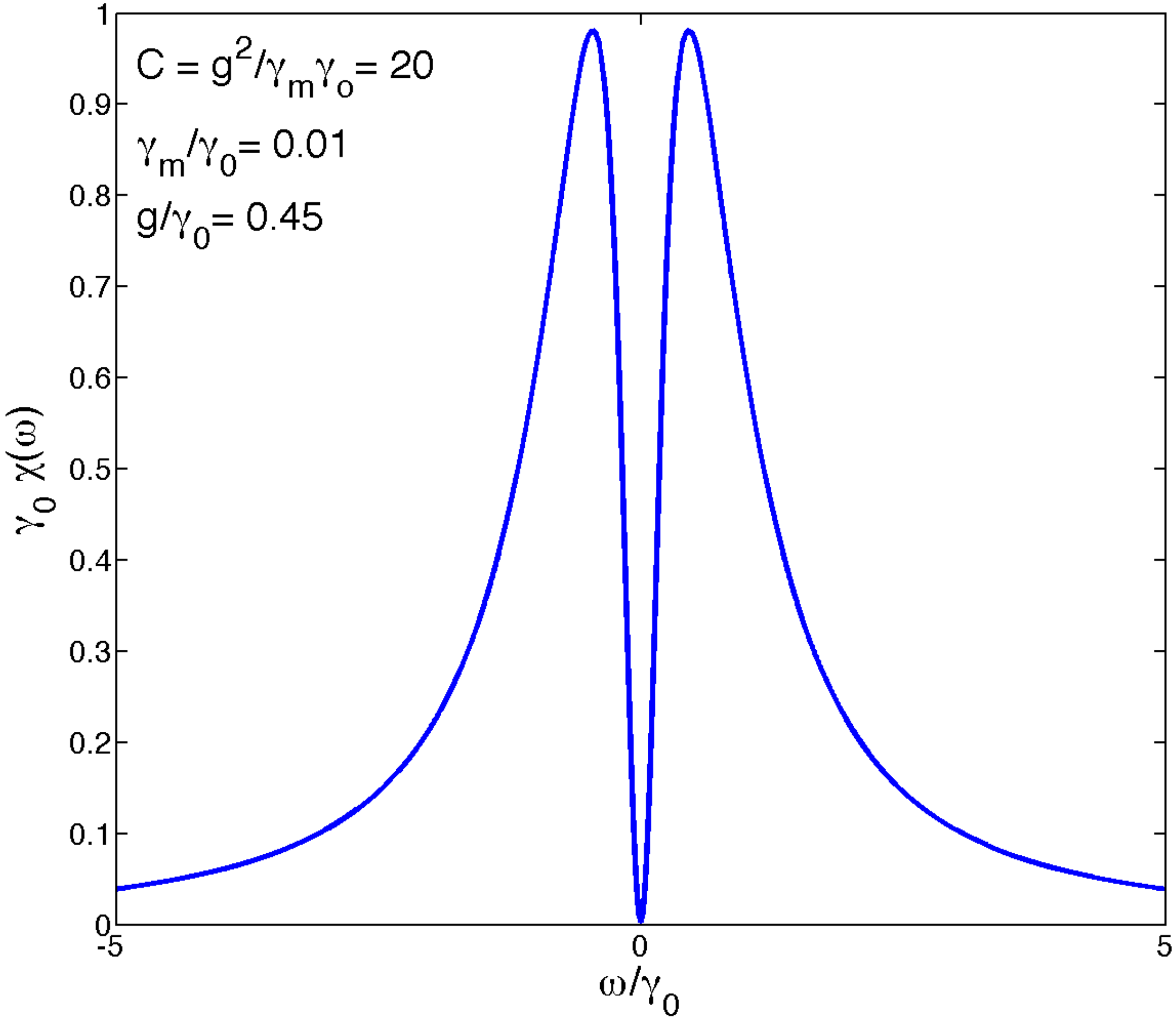


Optomechanically induced transparency

Optical susceptibility χ



Optomechanically induced transparency

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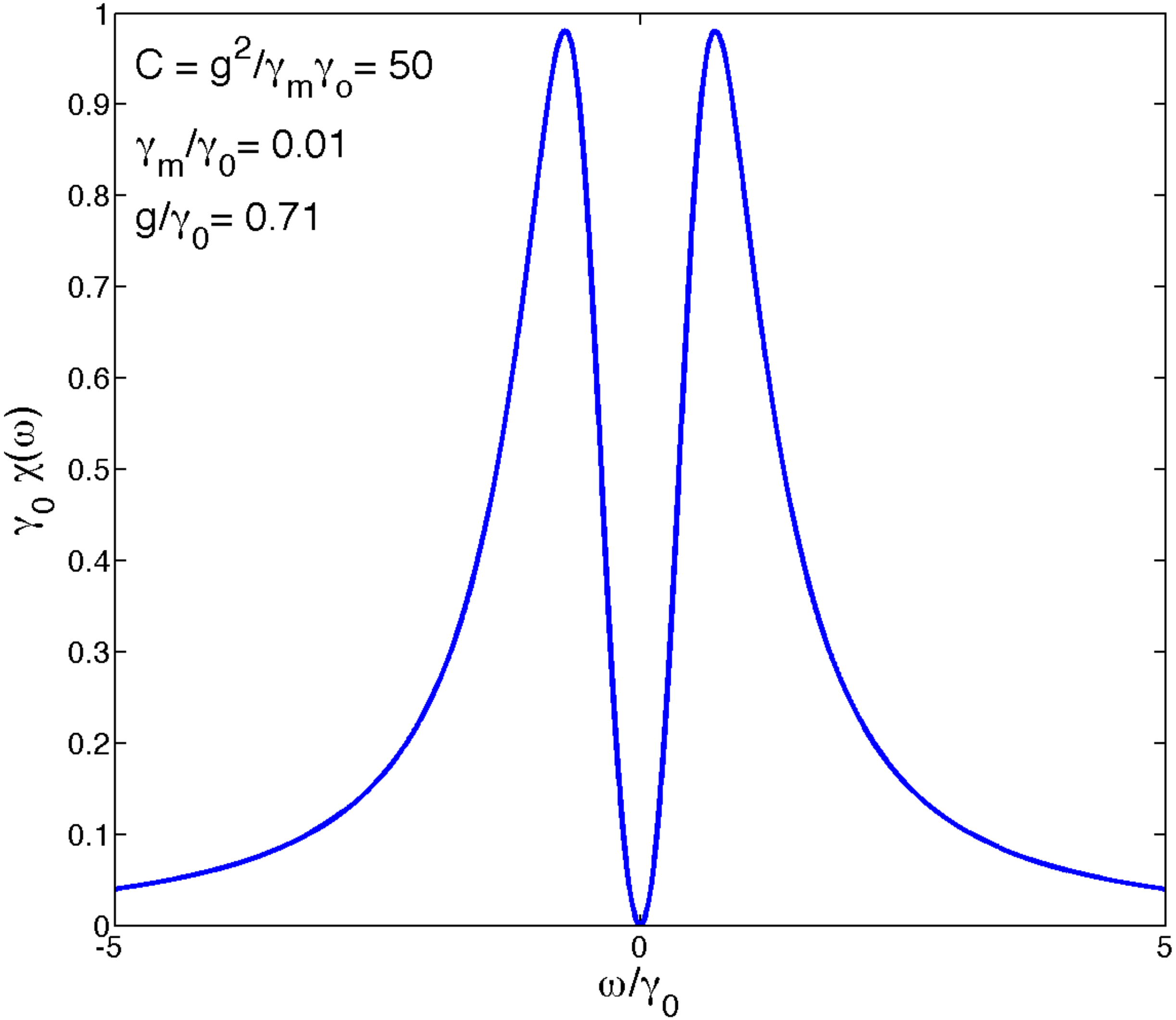

Optomechanically induced transparency

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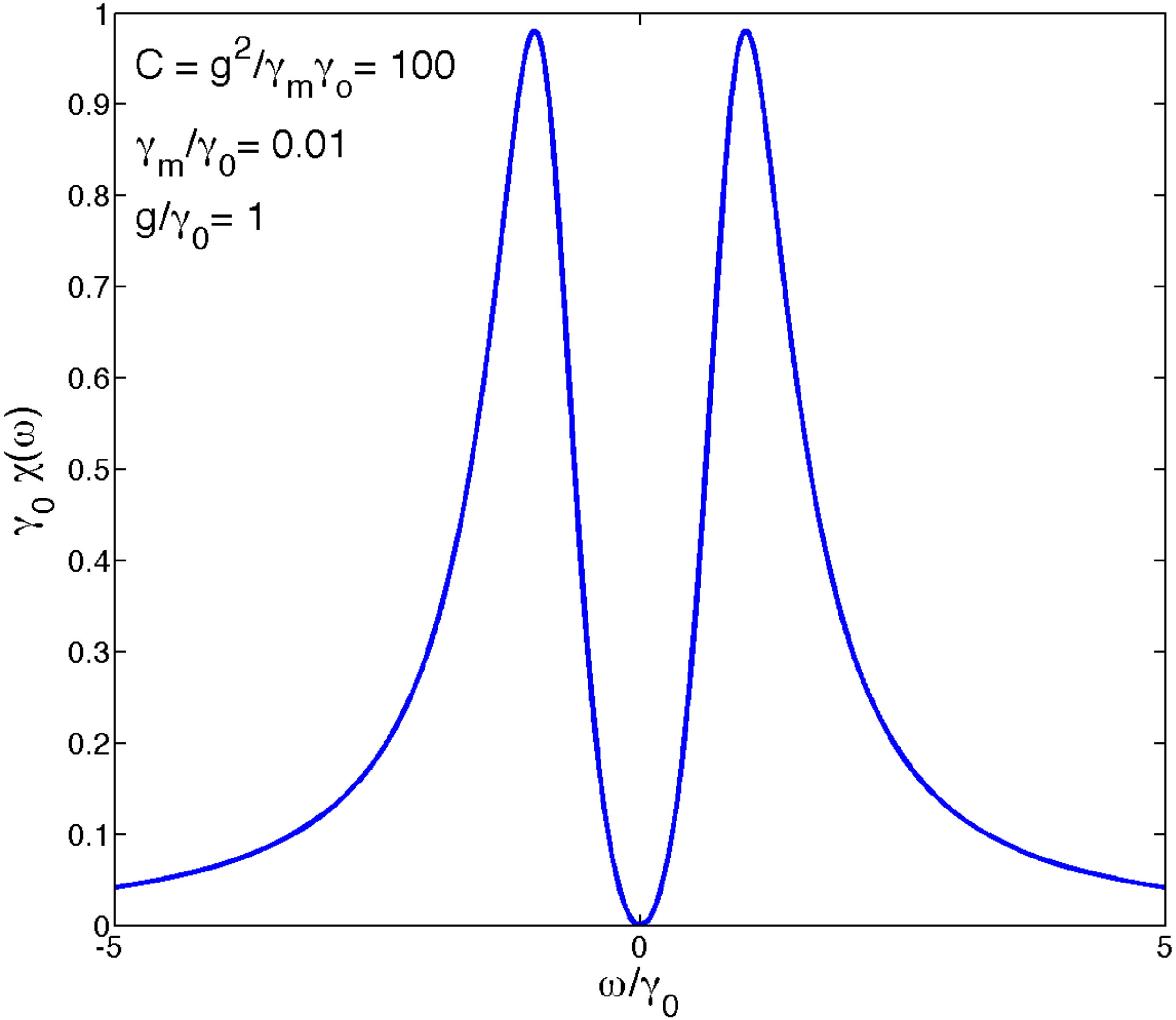

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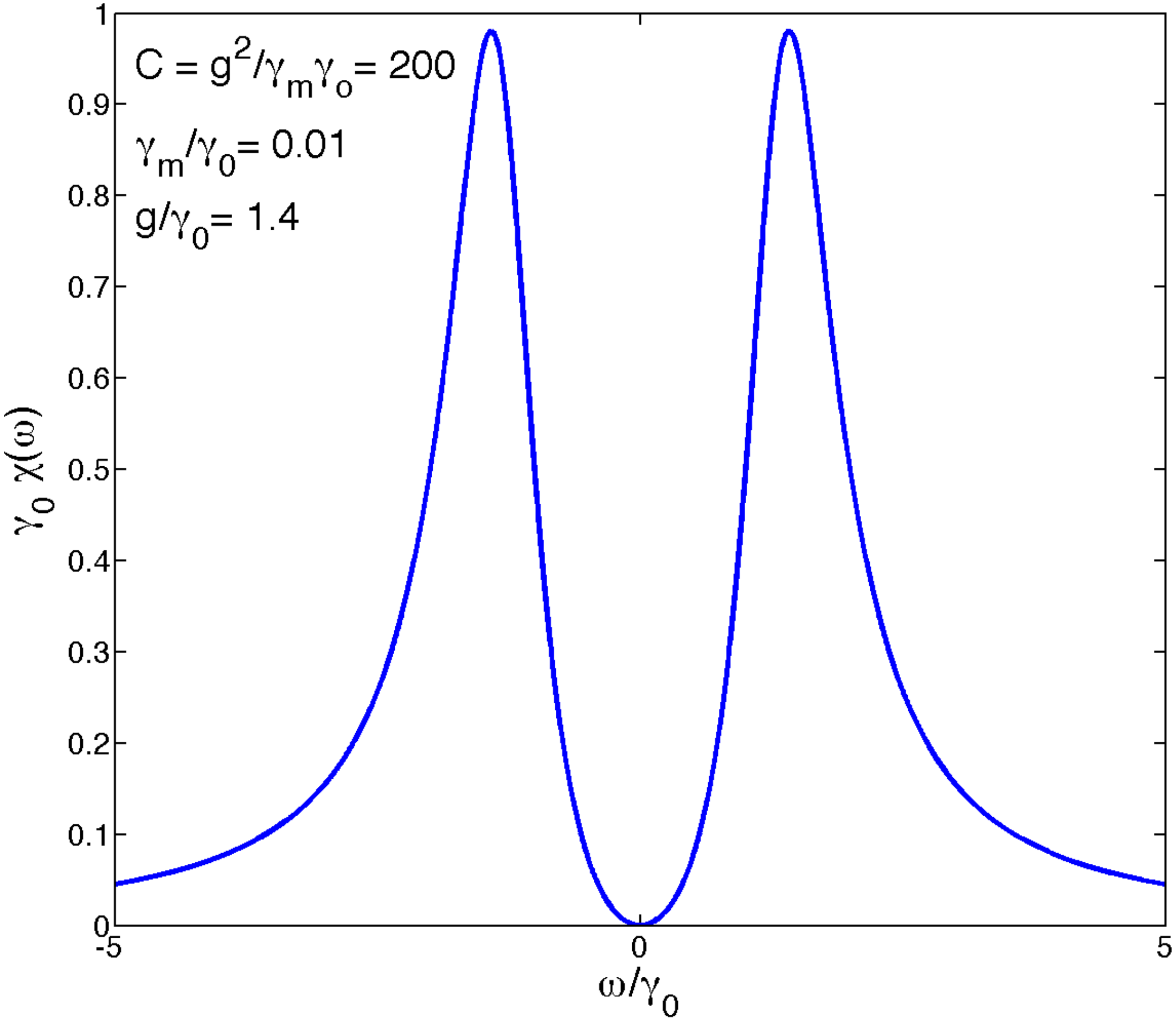
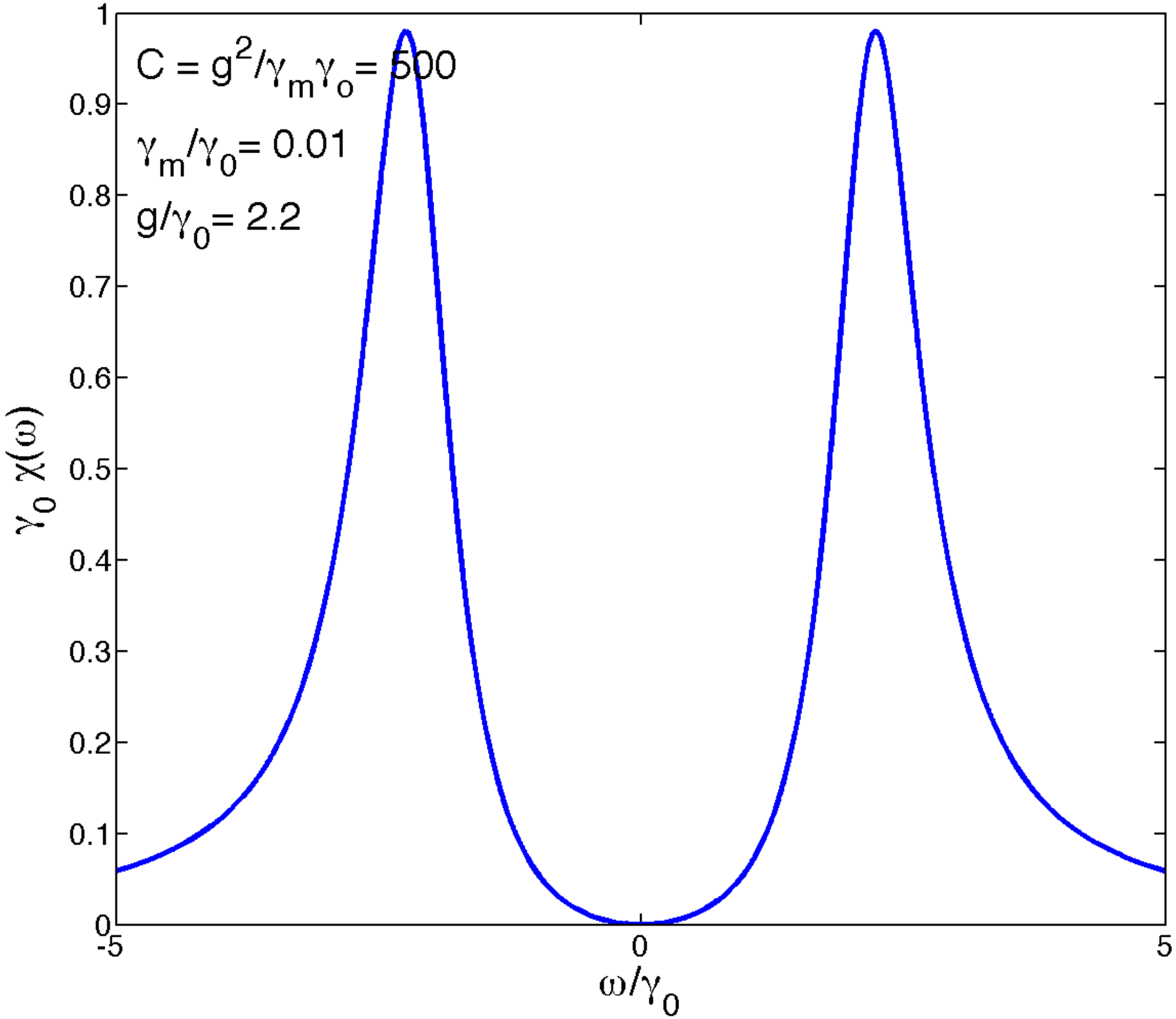
Optical susceptibility χ



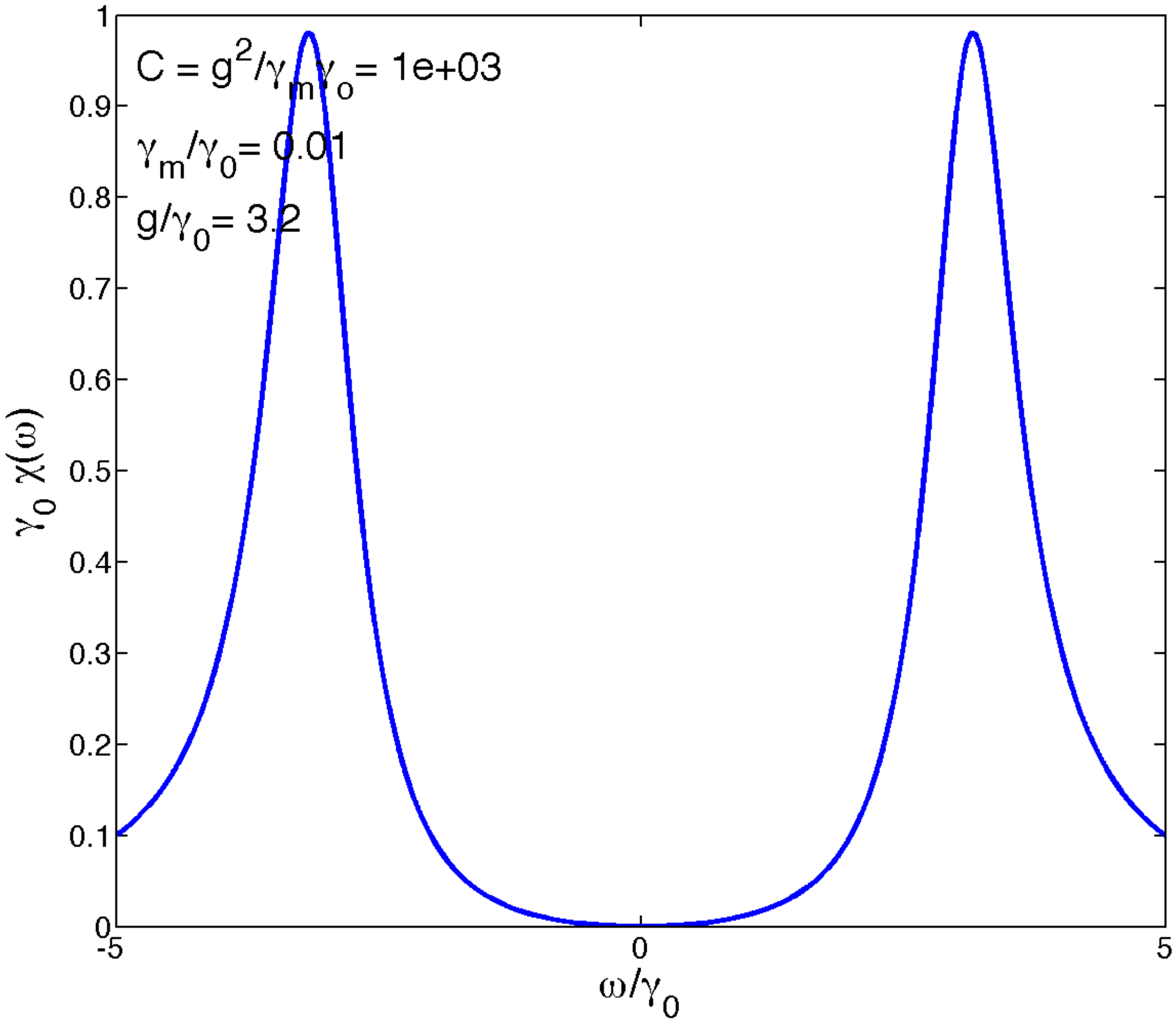
Optomechanically induced transparency

Optical susceptibility χ



Optomechanically induced transparency

Optical susceptibility χ



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Optical susceptibility at zero detuning (ω=0)

$$\chi(0)^{-1} = \gamma_o + \frac{g^2}{\gamma_m} = \gamma_o(1+C) \qquad C = \frac{g^2}{\gamma_m \gamma_o}$$

- $\chi(0)$ ➔ 0 as C ➔ ∞, light can no longer enter cavity!

Optomechanical modification when $\omega \ll \gamma_0$

$$\chi(\omega) = \chi_{g=0}(\omega) + \chi_{\mathrm{OM}}(\omega)$$

$$\chi_{\mathrm{OM}}(\omega) = \chi(\omega) - \chi_{g=0}(\omega)$$

$$= \frac{1}{\gamma_o \cancel{- i\omega} + \frac{g^2}{\gamma_m - i\omega}} - \frac{1}{\gamma_o \cancel{- i\omega}}$$

$$= \boxed{-\frac{1}{\gamma_o}\left[\frac{C\gamma_m}{\gamma_m(1+C) - i\omega}\right]}$$

Short exercise: derive this expression

A Lorentzian

Optomechanically induced transparency

Optical susceptibility at zero detuning (ω=0)

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$$= \frac{1}{\gamma_o \cancel{- i\omega} + \frac{g^2}{\gamma_m - i\omega}} - \frac{1}{\gamma_o \cancel{- i\omega}}$$

$$= -\frac{1}{\boxed{\gamma_o}}\left[\frac{\boxed{C\gamma_m}}{\boxed{\gamma_m(1+C)} - i\omega}\right]$$

Height

Short exercise: derive this expression

Optomechanically induced transparency

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Optical susceptibility at zero detuning ($\omega=0$)

$$\chi(0)^{-1} = \gamma_o + \frac{g^2}{\gamma_m} = \gamma_o(1+C) \qquad C = \frac{g^2}{\gamma_m \gamma_o}$$

- $\chi(0) \rightarrow 0$ as $C \rightarrow \infty$, light can no longer enter cavity!

Optomechanical modification when $\omega \ll \gamma_0$

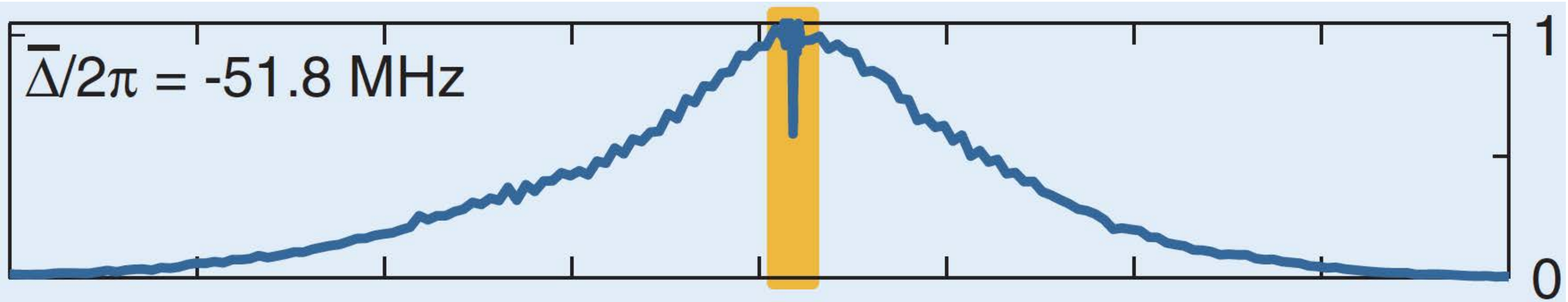
$$\chi(\omega) = \chi_{g=0}(\omega) + \chi_{\mathrm{OM}}(\omega)$$

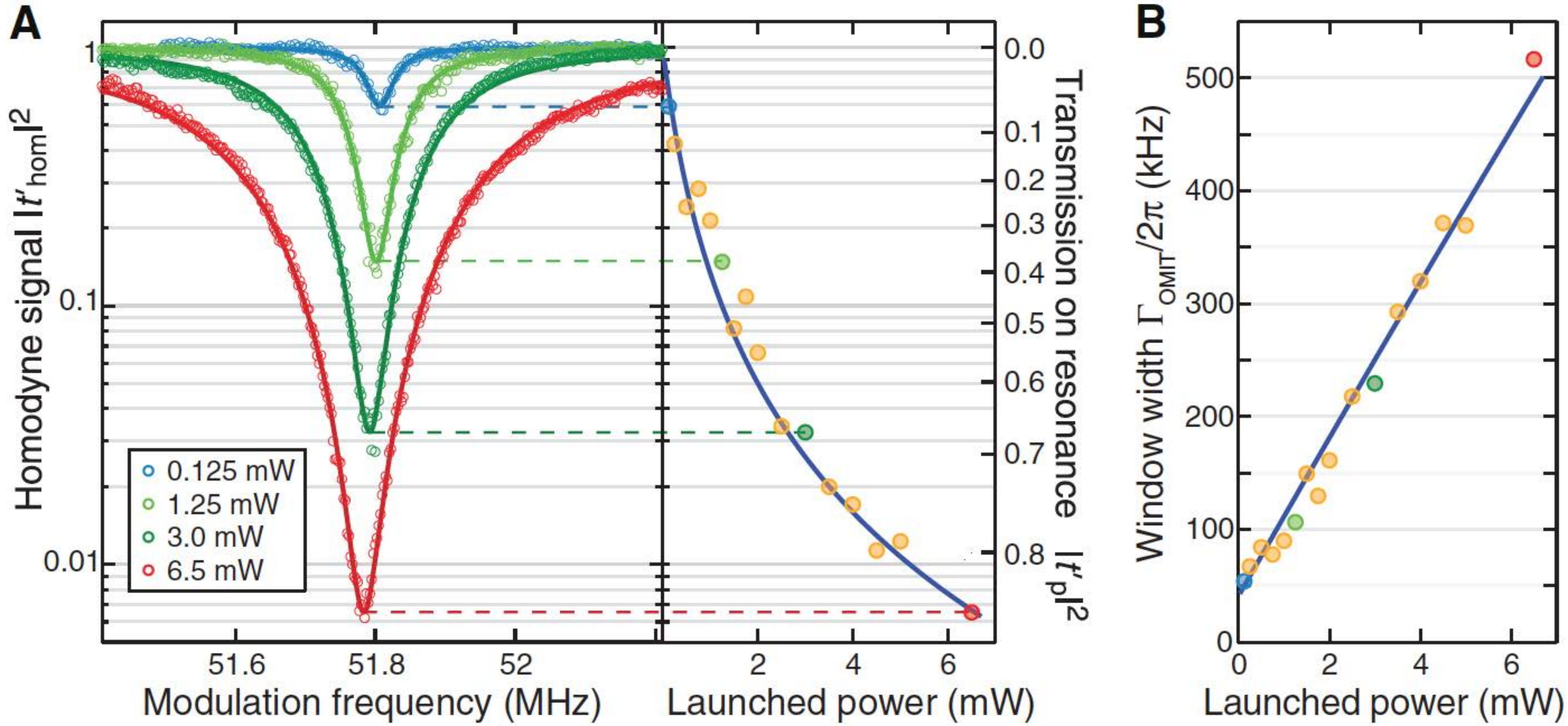
$$\begin{aligned}\chi_{\mathrm{OM}}(\omega) &= \chi(\omega) - \chi_{g=0}(\omega) \\ &= \frac{1}{\gamma_o \cancel{- i\omega} + \frac{g^2}{\gamma_m - i\omega}} - \frac{1}{\gamma_o \cancel{- i\omega}} \\ &= -\frac{1}{\gamma_o}\left[\frac{C\gamma_m}{\boxed{\gamma_m(1+C)} - i\omega}\right]\end{aligned}$$

Width

Short exercise: derive this expression

Optomechanically induced transparency





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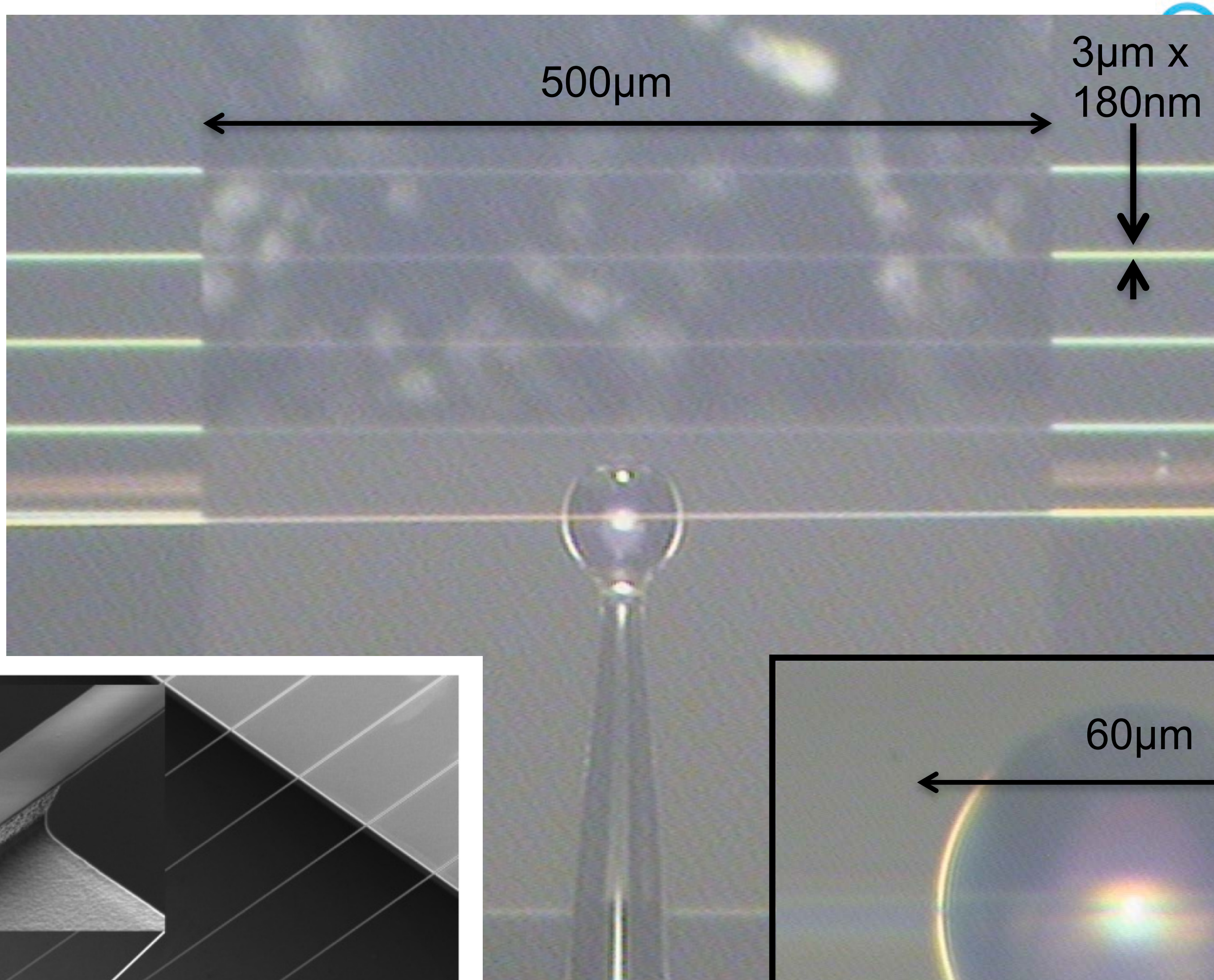


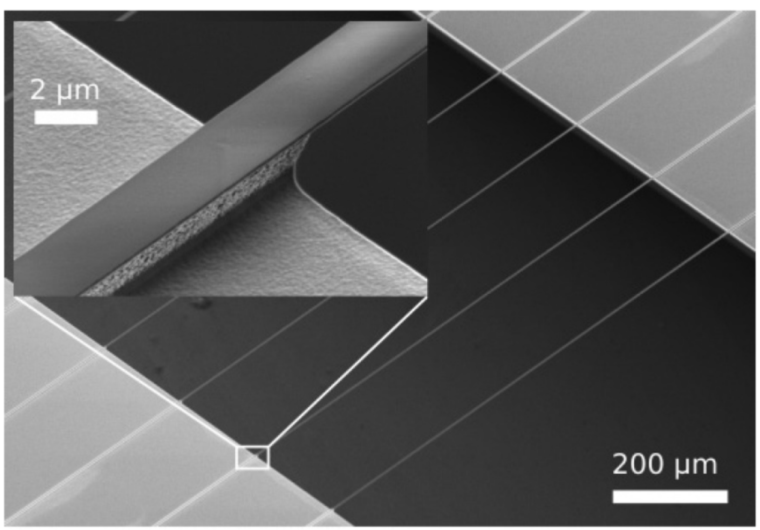
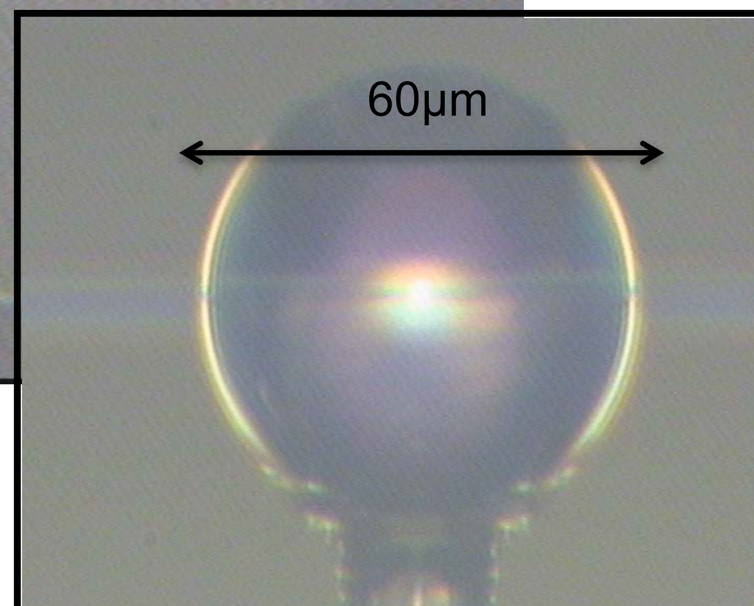
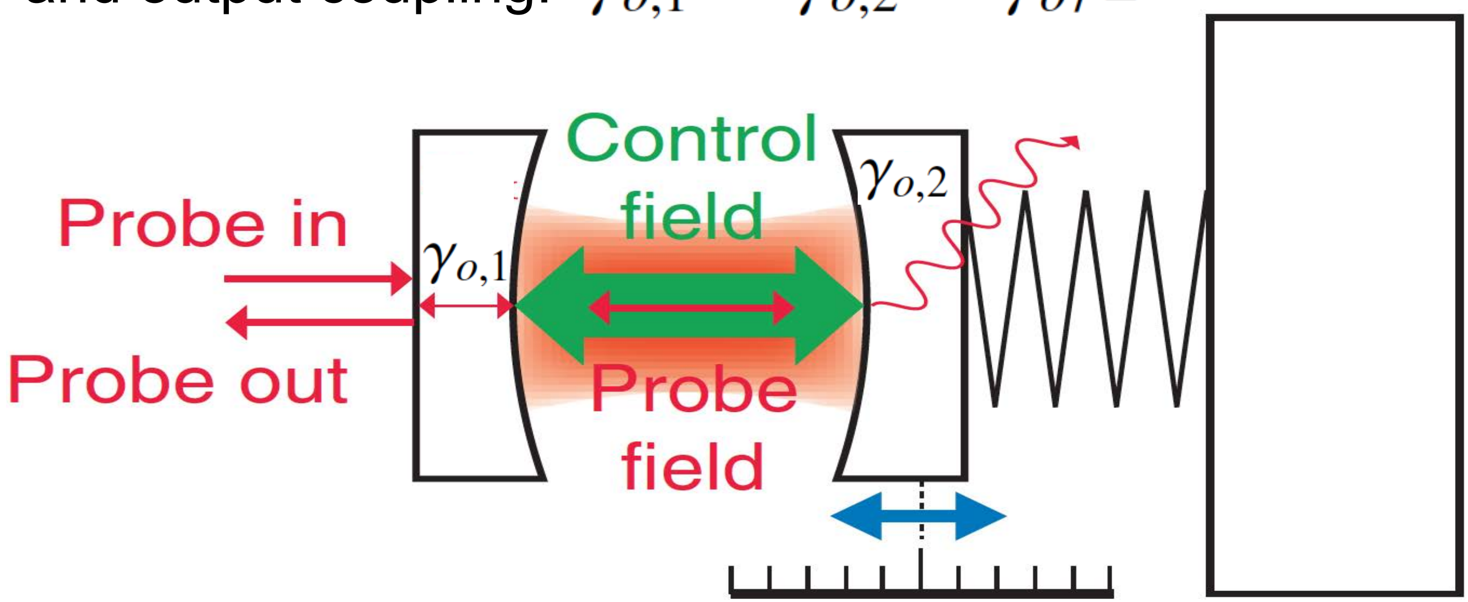


FIG. 2. SEM image of 157-nm thick, 3-μm wide high-stress silicon nitride strings.



Optomechanically induced transparency

- So far looked at intracavity field
- Ultimately measure (and are interested in) output field
- To observe transparency require that cavity is "impedance matched" and therefore perfectly absorbing without optomechanical interaction
- Achieved when cavity decay is split equally between loss and output coupling: $\gamma_{o,1} = \gamma_{o,2} = \gamma_o/2$



Input/output theory

- Optical fluctuations then enter the cavity equally through the input coupler and the loss port

$$\sqrt{2\gamma_o}\hat{a}_{\rm in} = \sqrt{\gamma_o}\hat{a}_{\rm in,1} + \sqrt{\gamma_o}\hat{a}_{\rm in,2}$$

- In an open quantum system, the field output through a port in the system is generally given by

$$\hat{a}_{\rm out} = \boxed{\hat{a}_{\rm in}} - \boxed{\sqrt{2\gamma}\hat{a}}$$

Incident field reflected from cavity mirror

Intracavity field transmitted through cavity mirror

- So in our specific case

$$\hat{a}_{\rm out,1} = \hat{a}_{\rm in,1} - \sqrt{\gamma_o}\hat{a}$$

Optomechanically induced transparency

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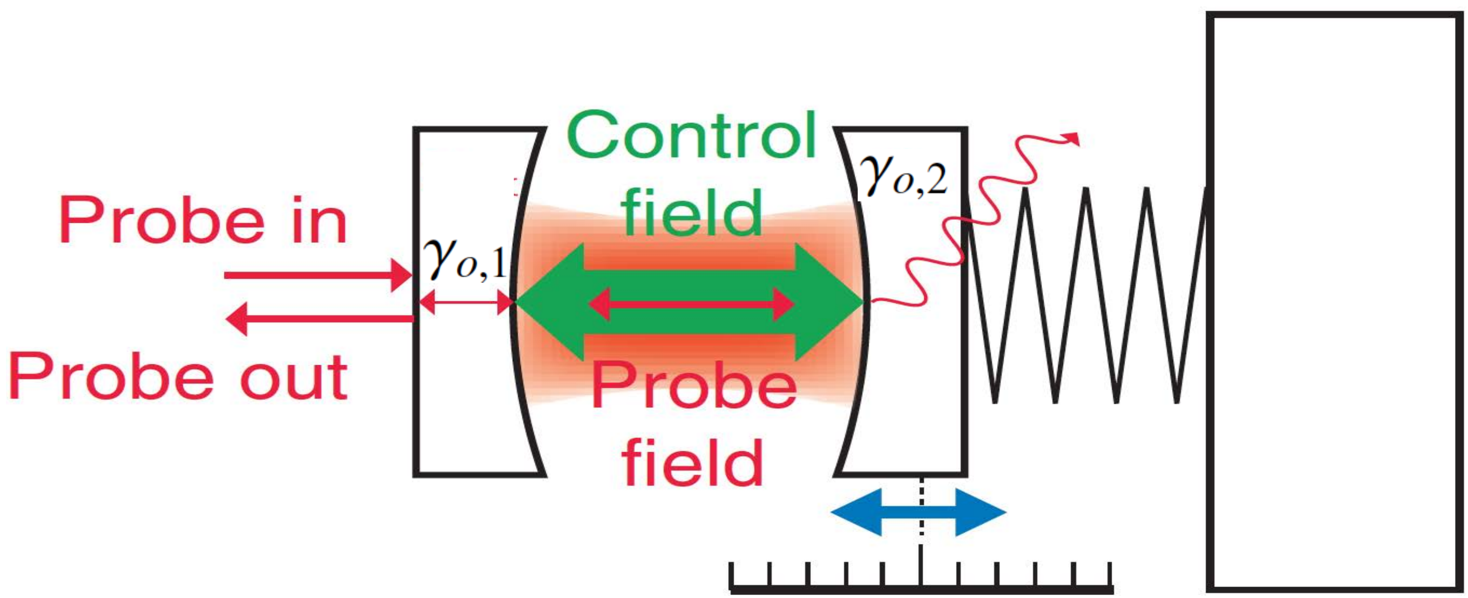
Key questions:

- What is the frequency response of the output field (i.e. the output optical susceptibility)?
- How significant are the fluctuations entering the output field through loss and the mechanical oscillator?, and, can they be suppressed with sufficiently high cooperativity (*C*)?



Output optical susceptibility

- …after some work, output field:

$$\hat{a}_{\text{out},1} = \boxed{[1-\gamma_o\chi(\omega)]}\,\hat{a}_{\text{in},1} + \gamma_o\chi(\omega)\left[\sqrt{2C}\left(\frac{i\gamma_m}{\gamma_m - i\omega}\right)\hat{b}_{\text{in}}(\omega+\omega_m) - \hat{a}_{\text{in},2}\right]$$

Output optical susceptibility χ_{out}

- Output optical susceptibility near the cavity resonance:

$$\chi_{\text{out}}(\omega \ll \gamma_0) = -\gamma_o\chi_{\text{OM}}(\omega \ll \gamma_0)$$

- We showed earlier that near resonance χ_{OM} is an negative Lorentzian.
- So near resonance χ_{out} is a positive Lorenztian – the cavity no longer absorbs the field, and becomes transparent.

Exercises: derive these results

Output quadrature variance

- Variance of arbitrary output quadrature given by:

$$\left\langle \left| X^{\phi}_{\mathrm{out},1}(\omega) \right|^2 \right\rangle = \left\langle \left| \hat{a}_{\mathrm{out},1}(\omega) e^{-i\phi} + \hat{a}_{\mathrm{out},1}(-\omega)^{\dagger} e^{i\phi} \right|^2 \right\rangle$$

- After some (more) work…

$$\left\langle \left| X^{\phi}_{\mathrm{out},1}(\omega) \right|^2 \right\rangle = |\chi_{\mathrm{out}}(\omega)|^2 \left\langle \left| X^{\xi}_{\mathrm{in},1}(\omega) \right|^2 \right\rangle + \frac{2\gamma_o^2 C |\chi(\omega)|^2 \gamma_m^2}{\gamma_m^2 + \omega^2} \left\langle \left| X^{\theta}_{b}(\omega) \right|^2 \right\rangle + \gamma_o^2 |\chi(\omega)|^2 \left\langle \left| X^{\zeta}_{\mathrm{in},2}(\omega) \right|^2 \right\rangle$$

where, ξ, θ, and ζ are phase angles which can be analytically found, but for phase insensitive noise sources (such as vacuum and thermal noise), do not matter.

- Coherent state input ➔ $\left\langle \left| X^{\xi}_{\mathrm{in},1}(\omega) \right|^2 \right\rangle = \left\langle \left| X^{\zeta}_{\mathrm{in},2}(\omega) \right|^2 \right\rangle = 1$, $\left\langle \left| X^{\theta}_{b}(\omega) \right|^2 \right\rangle = 2n + 1$
- On resonance (ω=0) in the limit that $n \gg 1$, find that
  - Optical loss term negligible if $\boxed{C \gg 1}$ **Cavity efficiently reflects *both* input fields**

Output quadrature variance

- Variance of arbitrary output quadrature given by:

$$\left\langle \left| X^{\phi}_{\mathrm{out},1}(\omega) \right|^2 \right\rangle = \left\langle \left| \hat{a}_{\mathrm{out},1}(\omega) e^{-i\phi} + \hat{a}_{\mathrm{out},1}(-\omega)^{\dagger} e^{i\phi} \right|^2 \right\rangle$$

- After some (more) work…

$$\left\langle \left| X^{\phi}_{\mathrm{out},1}(\omega) \right|^2 \right\rangle = |\chi_{\mathrm{out}}(\omega)|^2 \left\langle \left| X^{\xi}_{\mathrm{in},1}(\omega) \right|^2 \right\rangle + \frac{2\gamma_o^2 C |\chi(\omega)|^2 \gamma_m^2}{\gamma_m^2 + \omega^2} \left\langle \left| X^{\theta}_{b}(\omega) \right|^2 \right\rangle + \gamma_o^2 |\chi(\omega)|^2 \left\langle \left| X^{\zeta}_{\mathrm{in},2}(\omega) \right|^2 \right\rangle$$

where, ξ, θ, and ζ are phase angles which can be analytically found, but for phase insensitive noise sources (such as vacuum and thermal noise), do not matter.

- Coherent state input ➔ $\left\langle \left| X^{\xi}_{\mathrm{in},1}(\omega) \right|^2 \right\rangle = \left\langle \left| X^{\zeta}_{\mathrm{in},2}(\omega) \right|^2 \right\rangle = 1$, $\left\langle \left| X^{\theta}_{b}(\omega) \right|^2 \right\rangle = 2n + 1$
- On resonance (ω=0) in the limit that $n \gg 1$, find that
  - Optical loss term negligible if $C \gg 1$
  - Mechanical term negligible if $\boxed{C \gg 2\,(2n+1)}$

Condition for resolved sideband cooling to ground state

Exercises: derive these results too

What I hope you learnt

- How to use the quantum Langevin equations to predict the dynamics of basic experimentally relevant open quantum systems.
- How to use the rotating wave approximation to simplify a system Hamiltonian.
- How resolved resolved sideband cooling works in optomechanics.
- How optomechanically induced transparency works
- Some recent experimental progress in quantum optomechanics.

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